# Market variance and long-run asset pricing

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# Introduction

- A rather simplistic sentence often attributed to Keynes: "*In the long run, we are all dead*."
- Renewed attention to the long run in the industry (and in academia).
- In March 2010, **EFAMA** (European Fund and Asset Management Association) produced a report revisiting the landscape of European long-term savings.
- The goal of the report was to enable European citizens to take personal charge of their long-term savings in times of high economic uncertainty and longevity risk.

# Introduction

## Continued

Some of **EFAMA**'s recommendations:

1. Governments should introduce mandatory long-term saving schemes (with opt-out clauses) and propose suitable investment vehicles for implementation.
2. To mitigate risk when preparing for retirement, they recommend an automatic reduction over time of the share of risky assets in the portfolio.
3. To limit investors' incentives to opt-out, they suggest granting important tax advantages to mandatory long-term saving schemes.

These recommendations have spurred some debate. **Edhec-Risk** (Financial Times, April 19), for instance, has argued that while the proposal is sound, improvements can be made along a variety of dimentions. For example,

1. A deterministic glide path towards reducing risk cannot be justified in an optimal allocation framework. Risk attitudes need to be taken into account.
2. Given the state of public finances, a superior incentive would be for pension funds to offer a better risk-return trade-off through improved management.

# Long-run pricing

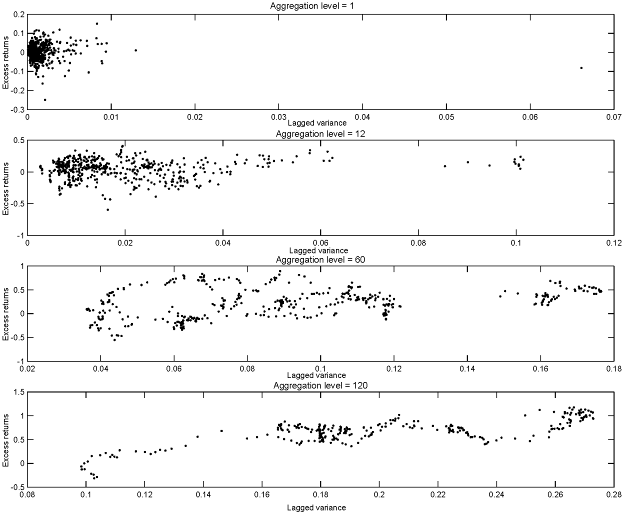
## Some issues

1. What drives market returns in the long run?
2. Does the variance of market returns affect market returns? In other words, is there a compensation for aggregate risk (as represented by market variance) at the market level?
3. How do classical cross-sectional pricing paradigms fare in the long run?

# Long-run risk-return trade-offs

- The traditional empirical evidence is ambivalent about the existence of a relation between market returns and market variance. This evidence focuses on *short horizons* (a month or a quarter).
- Bandi and Perron (Journal of Econometrics, 2008) show that, in the long run, the relation between market variance and (excess) market returns becomes "cleaner." In particular, long-run past market variance (*pmv*) has predictive ability for future long-run excess market returns.
- This predictive ability is superior to that of well-known predictors, such as the dividend-to-price ratio or the consumption-to-wealth ratio (Lettau and Ludvingson, Journal of Finance, 2001).

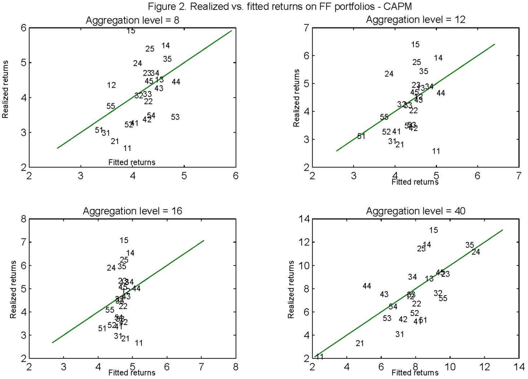
# Risk-return trade-offs at different horizons



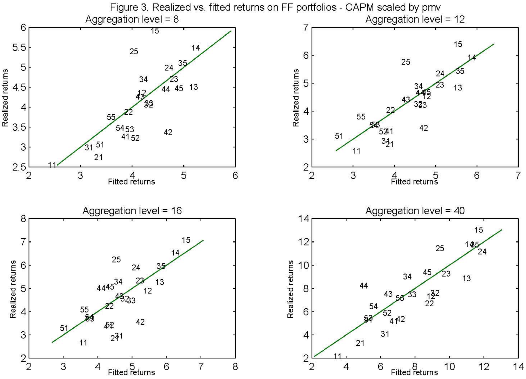
# Long run and the cross-section of stock returns

- We have focused on the market as a whole.
- *Does the long run have implications for individual stocks and portfolios as well?*
  1. Garcia and Lioui (Working paper, 2010) find that, while failing to explain cross-sectional stock returns in the short run, the popular Capital Asset Pricing Model (CAPM) "prices" the cross-section of stock returns more satisfactorily in the long run.
  2. Bandi and Perron (Working paper, 2009) show that computing long-run market betas (or betas with respect to consumption growth) conditional on past market variance (*pmv*) further reduces the pricing errors delivered by the CAPM (or the consumption-CAPM).
- Before providing details on the pricing models, we illustrate both observations graphically.

# CAPM at different horizons



# Conditional (on *pmv*) CAPM at different horizons



# Pricing models

## Intuition on cross-sectional pricing with one factor

- For every asset $p$, cross-sectional asset pricing implies that: $\mathbf{E}(R^p_{t,t+h} - R) = \lambda_f \beta^{R^p,f}$.
- The beta is a "quantity of risk" obtained from the regression:

$$R^p_{t,t+h} - R = \kappa + \beta^{R^p,f} f_{t,t+h} + \varepsilon_{t,t+h}$$

- Assume *risk varies with the state of the economy*, $\beta^{R^p,f}_t = b^p_1 + b^p_2 z_t$ (and $\kappa_t = a^p_1 + a^p_2 z_t$), then:

$$\begin{aligned} R^p_{t,t+h} - R &= (a^p_1 + a^p_2 z_t) + (b^p_1 + b^p_2 z_t) f_{t,t+h} + \varepsilon_{t,t+h} \\ &= a^p_1 + a^p_2 z_t + b^p_1 f_{t,t+h} + b^p_2 z_t f_{t,t+h} + \varepsilon_{t,t+h} \end{aligned}$$

- The cross-sectional model is now:

$$\mathbf{E}(R^p_{t,t+h} - R) = \lambda_z \beta^{R^p,z} + \lambda_f \beta^{R^p,f} + \lambda_{zf} \beta^{R^p,zf}.$$

# Pricing models

We consider several pricing models (several choices of $f_{t,t+h}$ and/or $z_t$):

- Unconditional models:
  1. CAPM: market returns as factor ($f_{t,t+h} = R^M_{t,t+h}$)
  2. C-CAPM: consumption growth as factor ($f_{t,t+h} = \Delta c_{t,t+h}$)
  3. Fama-French 3 factor model: returns on the market, SMB (returns on small firms minus returns on big firms), HML (returns on value firms minus returns on growth firms)
- Conditional models:
  1. CAPM with either $z_t = pmv_{t,t-h}$ or $z_t = cay_t$ (the consumption-to-wealth ratio) as scaling variable
  2. C-CAPM with either $z_t = pmv_{t,t-h}$ or $z_t = cay_t$ (the consumption-to-wealth ratio) as scaling variable

# Estimating pricing models in a nutshell

- Two-step procedure.
- Suppose we have 3 betas associated with $f_{t,t+h}$, $z_t$, and $z_t f_{t,t+h}$.

1. **First step**. For each portfolio $p$, we use a time-series regression to estimate the betas (the quantities of risk):

$$R^p_{t,t+h} = \kappa + \beta^{R^p,z} z_t + \beta^{R^p,f} f_{t,t+h} + \beta^{R^p,zf} z_t f_{t,t+h} + \varepsilon_{t,t+h}.$$

2. **Second step**. We regress cross-sectionally the average portfolio returns on the betas to estimate the lambdas (the prices of risk):

$$\left( \frac{1}{T-h} \sum_{t=1}^{T-h} R^p_{t,t+h} \right) = \alpha + \widehat{\beta}^{R^p,z} \lambda_z + \widehat{\beta}^{R^p,f} \lambda_f + \widehat{\beta}^{R^p,zf} \lambda_{zf} + u_{t,t+h}$$

- After discussing the data, we report a classical measure of pricing accuracy (the adjusted-$R^2$) for the second stage cross-sectional regressions.

# Data

- Quarterly data: 1952:2 - 2006:4.
- Cross-sectional asset returns: returns on the 25 Fama-French portfolios sorted on size and book-to-market (daily, aggregated to a quarter and longer periods).
- Market returns: returns on the CRSP NYSE-Amex-Nasdaq value-weighted portfolio with dividends (daily, aggregated to a quarter and longer periods).
- Consumption: real per-capita consumption on non-durables and services.
- Consumption-to-wealth ratio ($cay$): residuals from a regression of consumption on wealth and labor income. (Lettau and Ludvingson, Journal of Political Economy, 2001).

# Data
## Continued

- *Quarterly returns*: $R_{t,t+1} = \prod_{j=1}^{n_t}(1 + R_{t+\frac{j}{n_t}}) - 1$,
  where $n_t$ is the number of trading days in quarter $t$.
- *Quarterly market variance* is obtained by summing squared continuously-compounded daily market returns over each quarter: $\sigma^2_{t,t+1} = \sum_{j=1}^{n_t} r^2_{t+\frac{j}{n_t}}$
- The $h$-period returns are

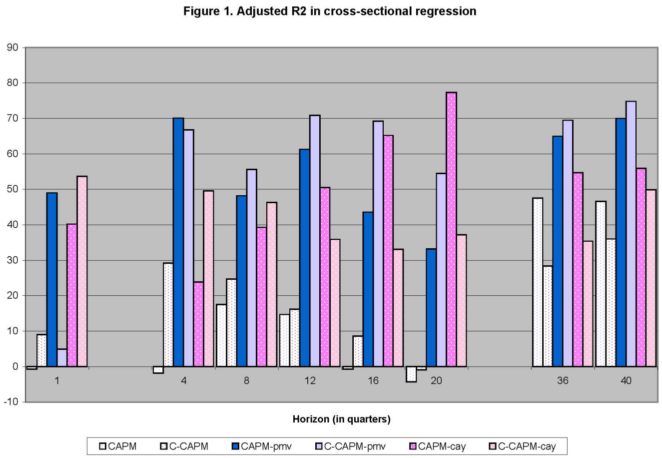
$$R_{t,t+h} = \prod_{j=1}^{h}(1 + R_{t+j}) - 1,$$

and the $h$-period market variance is

$$\sigma^2_{t,t+h} = \sum_{i=1}^{h} \sigma^2_{t+i-1,t+i} = pmv_{t,t+h},$$

for $h$ between 1 quarter and 40 quarters (10 years).

# Cross-sectional pricing errors at different horizons



# C-CAPM at different horizons

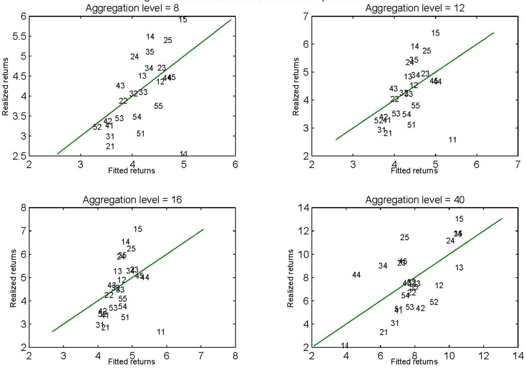
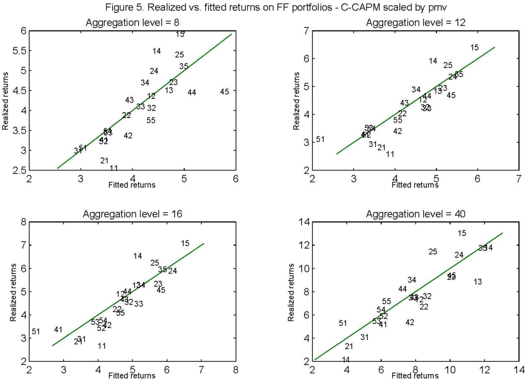


Figure 4. Realized vs. fitted returns on FF portfolios - C-CAPM

# Conditional (on *pmv*) C-CAPM at different horizons



# Comments

- Market returns (CAPM) and consumption growth (C-CAPM) become more effective factors as we move to longer horizons. This point has been recently made by Garcia and Lioui (Working paper, 2010) and Bandi and Perron (Working paper, 2009).
- Garcia and Lioui (2010) emphasize that the CAPM pricing errors may still be sizeable, even in the long run.
- Conditioning on either *cay* or *pmv* further improves the long-run fit (Bandi and Perron, 2009).
- *pmv* compares favorably with *cay* both at business-cycle frequencies and in the very long run.

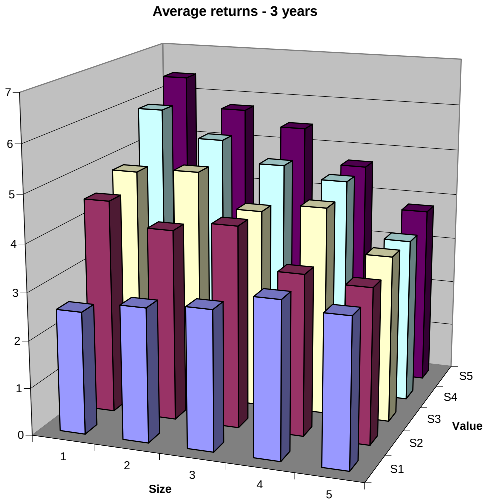
# Why conditioning? An economic interpretation

## Using the consumption-CAPM

- Small firms have higher average excess returns than large firms
- Value firms have higher average excess returns than growth firms (value premium)
- For the C-CAPM to yield small pricing errors, the betas with respect to consumption growth ought to align with average returns
- In other words, stocks yielding higher average returns should be riskier and therefore should have relatively larger betas (more "covariance") with consumption growth
- *This does not happen in the short run. It happens more in the long run. Conditioning, however, improves matters.*
- **Why scaling by pmv works?**

# The scaled (by *pmv*) consumption-CAPM model - 3 years

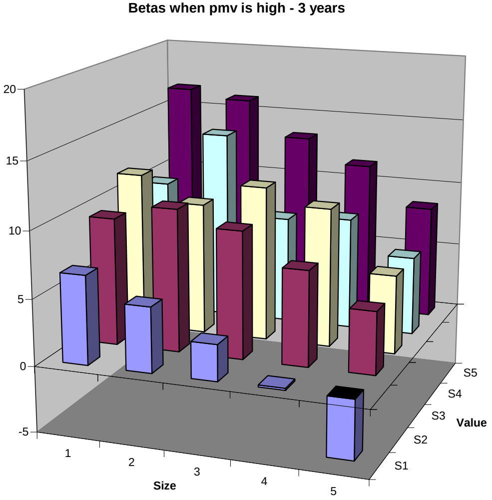
## A look at the average returns on the Fama-French portfolios



# The scaled (by *pmv*) consumption-CAPM model - 3 years

## The time-varying consumption betas - High pmv

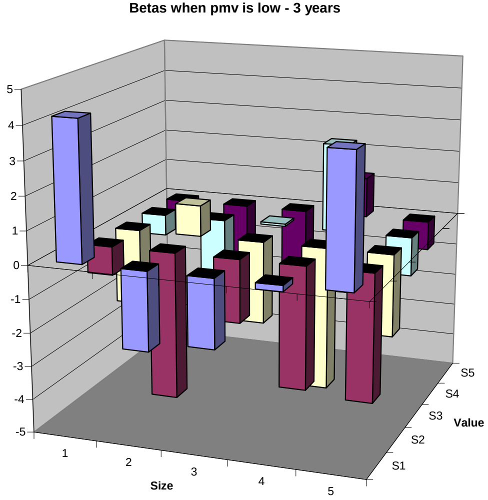
$$\beta_t^{R^p,\Delta c} = \widehat{\beta}^{R^p,\Delta c} + \widehat{\beta}^{R^p,\Delta c * pmv} pmv_{t-h,t}$$



# The scaled (by *pmv*) consumption-CAPM model - 3 years

## The time-varying consumption betas - Low pmv

$$\beta_t^{R^p,\Delta c} = \widehat{\beta}^{R^p,\Delta c} + \widehat{\beta}^{R^p,\Delta c * pmv} pmv_{t-h,t}$$



# Interpretation

- Small firms and value firms require higher average excess returns not because their *unconditional* risk (as measured by their *unconditional beta* with respect to consumption growth) is higher.
- Rather, they require higher average excess returns because their *conditional* risk (as measured by their *conditional - on the state of the economy - beta*) is higher in bad states of the world (i.e., when *pmv* is higher).

# Conclusions

- There is evidence of a market risk-return trade-off in the long run.
- Classical cross-sectional pricing paradigms fare better at lower frequencies.

Open questions:

1. Why does aggregation work? Is it a "signal extraction" mechanism? (Tamoni and Tebaldi, 2010.)
2. How do we think about the lung-run pricing problem in terms of stochastic discount factors? Should we really be thinking in terms of aggregation of short-term pricing models?