Investment Options with Debt Financing Constraints

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Abstract

We use a contingent claims model to study the impact of debt financing constraints on firm value, optimal capital structure, the timing of investment and other variables like the credit spreads. The optimal investment trigger follows a U-shape as a function of exogenously imposed constraint. Equity financed risky R&D growth options reduce the impact of debt constraints and increase firm value by increasing the option value on unlevered assets. We also investigate the socially optimum level of debt and the effect of financing constraints on government taxes and social welfare. Finally, a model of differential beliefs between debt and equity holders (about growth rate or volatility) is proposed that endogenizes debt constraints.

Introduction

Firms may face debt constraints for various reasons. Exogenous debt constraints may be due for example to the compliance to minimum capital requirements set to financial institutions. Frictions due to moral hazard or asymmetric information (see Jensen and Meckling, 1976 and Myers and Majluf, 1984) may also create debt constraints[1]. Asymmetric information can also justify why the suppliers of credit may engage in credit rationing (see for example Fazzari et al., 1988, Stiglitz and Weiss, 1981 and Pawlina and Renneboog, 2005). This study investigates the effect of debt financing constraints on firm value, the timing of investment and optimal default decision and other important variables like the credit spreads. We use a contingent claim approach incorporating risky pre-investment R&D options and also investigate the tax raising and social welfare implications of debt financing constraints. Lensik and Sterken (2002) use a real options approach without incorporating a stochastic model for debt and optimal capital structure and discuss conditions under which credit rationing by banks may apply. We analyze the effect of exogenous debt constraints and we then also endogenize debt constraints focusing on differential information between equity and debt holders with respect to the growth rate or volatility of the underlying asset.

Rauh (2006) and Hubbard et al. (1995) show empirical evidence that firms face financing constraints that are attributed to possible debt and equity market frictions. Whited and Wu (2006) and Gomes et al. (2006) document empirically the significance of financing constraints and show that they represent a risk factor of firm returns. Boyle and

[1] Debt constraints may also be the result of the internal policy of the firm to reduce the risk of potential bond downgrading or involuntary default.

Guthrie (2003) (see also Cleary et al., 2007) analyze the effect of liquidity constraints on investment decisions. Our emphasis is to explicitly consider the valuation of debt and adjustments from optimal (unconstrained) capital structure due to exogenous or endogenous debt financing constraints. We thus provide theoretical predictions using a structural model based on trade-off theory between tax benefits and bankruptcy costs. Other related work on financing constraints is that of Uhrig-Homburg (2004) that explores costly equity issue that can lead to a cash flow shortage restriction and Titman et al. (2004) that investigates the impact of financing constraints on default spreads but without modeling optimal capital structure.

Since Merton (1974) the contingent claim approach has been extended to the valuation of levered firms including the tax benefits of debt and bankruptcy costs (for example, Brennan and Schwartz, 1978, and Kane et al., 1984, and 1985). Leland (1994) uses a perpetual horizon assumption and derives closed form expressions for the value of levered equity, debt and the firm in the presence of taxes and bankruptcy costs analyzing equity holders optimal capital structure and default decisions. Leland and Toft (1996) extend Leland (1994) to allow the firm to choose the optimal maturity of the debt. Mauer and Sarkar (2005) include investment option decisions deriving closed form solutions[2]. Gamba and Triantis (2005) consider personal and corporate taxes, capital issuance costs

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[2] Other papers that incorporate the investment option decision are Brennan and Schwartz (1984) and Gamba et al. (2005). Mauer and Triantis (1994) use a dynamic model of capital structure with default based on bond covenant restrictions. Fries et al (1997) explore the valuation of corporate securities (debt and equity) incorporating the tax benefits, bankruptcy costs and the agency costs of debt in a competitive industry with entry and exit decisions. Valuation of corporate securities in a duopoly with entry and exit decisions has been studied by Lambrecht (2001). In this paper we do not explicitly model competition but we allow for exogenous competitive erosion.

and liquidity constraints in a dynamic model without endogenous (optimal) default determination

We build on Mauer and Sarkar (2005) framework and we incorporate exogenous and endogenous (due to differential information) debt constraints. We show that debt constraints reduce firm value more significantly at higher levels of competitive erosion, lower volatility of assets, higher tax rates and low bankruptcy costs--situations where the net benefits of debt are more important. Adjustments to meet the constraint also depend on the optimal debt capacity at the unconstrained level and the trade-offs between foregone investment timing flexibility and the net benefits of debt. These trade-offs create a U-shape of the investment trigger with respect to the level of debt financing constraints. Finite maturity horizon for the investment option results in a decreased firm value and a more pronounced effect of the constraint. R&D growth options reduce the important impact of debt financing constraints especially for lower maturity options by enhancing option value due to an increase in expected returns or increased volatility. We also show that R&D will increase firm value mostly by increasing the value of the option on the unlevered assets while their effect on the expected net benefits of debt is of lesser importance. We find that the exercise of R&D growth options decrease leverage ratios and expected credit spreads in the presence of constraints in contrast to the case of no constraints where R&D does not have an impact on leverage or credit spreads. We also study the effect of exogenous debt financing constraints on the level of taxes raised by the government and through a social welfare function that captures the representative firm's value and government taxes. Our results show that there may be an optimal level of debt constraints for the overall economy under certain model parameterizations reflecting a trade-off between firm value reduction and taxes increase.

In the last part of the paper we endogenize debt constraints by introducing differential beliefs on volatility and the growth rate of assets. Our results show that when debt holders estimate of the firm's volatility is higher or the growth rate of assets is lower than equity holders estimate, optimal leverage and firm values get reduced. These unfavourable beliefs of debt holders act as an endogenous constraint on the use of debt and create adjustments in the firm's investment policy and capital structure. An important difference relative to the exogenous constraint case is that we no longer observe a U-shape in the investment trigger with the firm delaying investment when endogenous debt constraints exist.

I. The model

Following Leland (1994) we assume that the firm's unlevered assets follow a Geometric Brownian Motion

$$\frac{dV}{V} = \mu dt + \sigma dZ \tag{1}$$

where μ denotes the capital gains of this asset, σ denotes its volatility, and dZ is an increment of a standard Weiner process. Similarly to Leland (1994) we assume that V is unaffected by the firm's capital structure: any coupon payments on debt are financed by new equity leaving the value of unlevered assets unaffected. We however allow a dividend-like opportunity cost of waiting to invest δ that may be used to capture competitive erosion on the value of assets (e.g., Childs and Triantis 1999, Trigeorgis 1996 ch.9, and Trigeorgis 1991). A low δ affects the (risk-neutral) drift $r-\delta$ used in the valuation showing that a low δ effectively increases the growth rate of the value of unlevered assets (see also McDonald and Siegel, 1984).

Figure 1 shows the sequence of decisions in our model. Equity holders have a first-stage R&D option to enhance the value of assets before full development. The exercise has an instantaneous impulse effect (no time-to-build). Its purpose is to enhance the value of unlevered assets but it has an uncertain outcome. The R&D option is fully characterized by its volatility, expected impact and cost and may represent product redesign, advertisement or other actions that are targeted towards an increase in value, albeit having an uncertain outcome. We wish to study the effect of such actions on firm value and its components (option on unlevered assets and the net benefits of debt), and on the expected optimal leverage, equity and debt value, and credit spreads.

[Insert Figure 1]

We assume that the R&D option can be exercised at time zero at a cost I_C that is all-equity financed. All-equity financing is a reasonable assumption for start-up growth firms. The R&D option will have a multiplicative random outcome ($1+k$) on the value of unlevered asset where:

$$\ln(1+k) \sim N\left(\gamma - \frac{1}{2}\sigma_C^2, \sigma_C^2\right). \tag{2}$$

The assumption of a lognormal distribution is convenient since we retain the lognormality of the asset values when growth options are exercised. The expected impact on V is $1+\bar{k}=\exp(\gamma)$ with a variance $\exp(\gamma)\left(\exp(\sigma_C^2)-1\right)^{0.5}$. We assume that an equilibrium continuous-time CAPM (see Merton, 1973) holds and that impulse-type growth options have firm-specific risks which are uncorrelated with the market portfolio and are thus not priced. This assumption can be relaxed by using equilibrium models of priced jump risk (e.g. Bates, 1991). In our case the jumps in asset values are endogenous through the optimal exercise of the R&D option. Impulse controls with uncertain outcome have been studied in Korn (1997). In general we may have multiple stages of R&D and issues of path dependency (see Koussis, Martzoukos, and Trigeorgis, 2007, for an all-equity model with growth options). For simplicity here we assume that R&D options are available only at *t* = 0.

Optimal firm value, *F**(*V*) is calculated as the option to invest capital I_C at time zero that will potentially enhance *V* but has a random outcome. This gives the investment option $F(V)$ to pay capital cost I and acquire a potentially levered position $V^L(V)=E(V)+D(V)$. Note that $E(V)$ and $D(V)$ denote the stochastic values of equity and debt respectively. Optimal firm value at *t* = 0 can be defined as follows:

$$F^*(V)=\max_{\varphi_{I_C}}\{E^C[F(V)]-I_C,F(V)] \tag{3}$$

where $\varphi_{I_C}=$ {exercise of R&D option, no exercise of R&D option} and $E^C[.]$ is expectation conditional on the exercise of R&D option. For the evaluation of this conditional expectation we use a Markov chain implementation creating a grid of V values with respective probabilities consistent with the distribution described in equation (2).

Following the decision to exercise or not the R&D option the current equity holders get the option to invest capital I that can be partially financed with borrowing. For this part and under the perpetual investment horizon assumption we maintain the analytic framework of Leland (1994) and Mauer and Sarkar (2005) for the value of the firm. We call this framework (with unconstrained debt financing) the Extended Leland/MS model. Our analysis then focuses on the constrained debt optimization problem which is the main emphasis of this paper. In the most part of our analysis we retain the perpetual investment horizon assumption; for some cases where we investigate the impact of a finite investment horizon we implement a numerical lattice where at each node we solve an optimization problem by maintaining the perpetual horizon for equity and debt and thus using the relevant analytic formulas on the tree.

Firm value $F(V)$ is wholly owned by current equity holders. Its value derives from the option to optimally select the time (t_I) of investment taking into consideration that it can be partially financed with debt $D(V)$. Equity holders will thus pay the investment costs, receive $D(V)$ (in cash) from debtholders, and get a levered equity position $E(V)$ (that also includes the option to default). The money the firm actually needs to pay (the equity financing, not to be confused with equity value) equals $I - D(V)$. Thus the current equity holders have the option on $\max(E(V) - (I - D(V)),0)$ which is equivalent to $\max(E(V) + D(V) - I,0)$.

Equity value conditional on investment and default at V_B equals (see also Leland, 1994, and Mauer and Sarkar, 2005):

$$E(V_I) = V_I - \frac{R}{r} + \tau \frac{R}{r} + \left[\left[\frac{R}{r} \right] - V_B - \left[\tau \frac{R}{r} \right] \right] \left[\frac{V_I}{V_B} \right]^{\beta}$$

$$\beta = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} < 0 \qquad (4)$$

The parameters τ, R, r we denote the tax rate, coupon, and the risk free rate respectively. Equity holders will obtain the value of unlevered assets V_I minus a perpetual stream of coupon payments (second term) plus the tax benefits (third term) plus the option to default saving the interest payments on debt by giving up the value of assets at default and the tax benefits from that point forward (last term).

Similarly to Leland (1994) and Mauer and Sarkar (2005) equation (5) below shows the value of debt *D*(*V*) when debt holders have full information about default risk and other parameters. Debt holders will account for foregone interest at default thus accounting for default risk in the determination of *D*(*V*). They will also take into consideration and proportional to *V* bankruptcy costs (defined by parameter *b*).

$$D(V_I) = \frac{R}{r} - \left[\frac{R}{r} \right] \left[\frac{V_I}{V_B} \right]^{\beta} + (1-b) V_B \left[\frac{V_I}{V_B} \right]^{\beta} \qquad (5)$$

At the investment trigger, equity holders would want to maximize their position, that is $E(V_I) + D(V_I) - I$. Combining equation (4) with (5) gives equity holders position at the investment trigger:

$$F(V_I) = (V_I - I) + \frac{\tau R}{r}\left(1 - \left(\frac{V_I}{V_B}\right)^{\beta}\right) - bV_B\left(\frac{V_I}{V_B}\right)^{\beta} \tag{6}$$

Firm value at the investment trigger equals the value of unlevered assets plus the expected value of tax benefits until default minus the expected value of bankruptcy costs.

As in Leland (1994) the optimal default trigger is[3]:

$$V_B = \frac{-\beta}{(1-\beta)}(1-\tau)\frac{R}{r} \tag{7}$$

Note that since $\beta < 0$, V_B is positive. The equity holders option to invest is given by:

$$F(V) = \left[E(V_I) + D(V_I) - I\right]\left(\frac{V}{V_I}\right)^{a} \text{ where}$$
$$a = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{8a}$$

Equivalently, 8a can be re-written as:

$$\begin{aligned} F(V) &= (V_I - I)\left(\frac{V}{V_I}\right)^{a} + \frac{\tau R}{r}\left(1 - \left(\frac{V_I}{V_B}\right)^{\beta}\right)\left(\frac{V}{V_I}\right)^{a} - bV_B\left(\frac{V_I}{V_B}\right)^{\beta}\left(\frac{V}{V_I}\right)^{a} \\ &= E[V-I] + E[TB] - E[BC] \end{aligned} \tag{8b}$$

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[3] As noted in Leland (1994, p. 1222) the value of V_B that maximizes equity holders firm value at investment should be as low as possible (effectively $V_B = 0$). This selection will set bankruptcy costs to zero and keeps the flow of tax benefits until infinity. Given this choice a high as possible coupon value would be optimal. As noted in Leland (1994) however, a low default trigger cannot hold since there is a bound on V_B so that equity $E(V)$ remains positive. Equation (4) shows that setting V_B = 0 and letting R be very high may cause equity values to become negative. This can be avoided by imposing a smooth pasting condition $\left.\frac{\partial E(V)}{\partial V}\right|_{V=V_B}$ where $V = V_I$ on equity value at default. Note also that the promise of no default is not credible since equity holders will have the incentive to choose V_B to maximize equity value once debt is issued. For this reason the default trigger should also be the one that maximizes equity value i.e., $\frac{\partial E(V)}{\partial V_B} = 0$, with $V = V_I$. It turns out that both conditions (the smooth-pasting condition and the equity maximizing condition) result in the same optimal default trigger shown above.

where E[.] in the last line stands for expectation. The last line effectively shows that the value of the firm can be written as the expected value of the unlevered assets (option on unlevered assets) plus the expected value of tax benefits minus the expected value of bankruptcy costs (as in Mauer and Sarkar, 2005, but with emphasis on the value of the unlevered assets). The net benefits of debt are defined as the difference between the expected tax benefits and the expected bankruptcy costs, i.e., $NB = E(TB) - E(BC)$. This decomposition will prove useful in discussing the effect of financing constraints since it shows that it will involve a trade-off between investment timing (option value) and the net benefits of debt.

V_I is selected to maximize current equity holders option value given by equation (8a) (or equivalently 8b). The first order condition is calculated by applying $\left.\frac{\partial F}{\partial V}\right|_{V=V_I} = \left.\frac{\partial V^L}{\partial V}\right|_{V=V_I}$ and is given in equation (9) below:

$$\begin{aligned}&1+\beta\left((1-\tau)\frac{R}{r}-V_B\right)\left(\frac{V_I}{V_B}\right)^{\beta}\left(\frac{1}{V_I}\right)+\\&\beta\left((1-b)V_B-\frac{R}{r}\right)\left(\frac{V_I}{V_B}\right)^{\beta}\left(\frac{1}{V_I}\right)-\alpha\left(\frac{1}{V_I}\right)\left(E(V_I)+D(V_I)-I\right)=0\end{aligned} \tag{9}$$

Mauer and Sarkar (2005) use this framework (without the R&D option component) to study agency issues between equity holders and debt holders. The condition above for the investment trigger is equivalent to their “first-best” condition of firm value maximization. We will call the above model the Extended-Leland/MS model. It includes Leland (1994) and McDonald and Siegel (1986) (McD&S thereon) as special cases.

At the time of investment the equity holders will select the optimal level of the coupon payment that determines optimal capital structure. It can be easily seen from the equation (6) that the coupon payment should be selected simultaneously with the investment trigger since both the coupon and the investment trigger affect firm's debt capacity and the risk of default (see the Appendix for solution details).

With debt financing constraints current equity holders need to solve the following constrained optimization problem:

$$
\begin{aligned}
&\max_{V_I, R} \quad F(V_I, R) = \left[\left(E(V_I, R) + D(V_I, R) - I \right) \left(\frac{V}{V_I} \right)^a \right] \\
&s.t. \\
&D(V_I, R) \le D^{\max} \\
&V_B = \frac{-\beta}{(1-\beta)} (1-\tau) \frac{R}{r}
\end{aligned}
\tag{10}
$$

The problem involves a non-linear objective function and a non-linear constraint. Under the assumption of a perpetual investment horizon we use the analytic formulas described above and solve the equity holders optimization problem through a numerical (dense grid) search for various coupon levels that satisfy the first order condition of the investment trigger until the constraint becomes binding (in which case we adjust the investment trigger to meet the constraint). Our approach is consistent with the "first-best" strategy for the firm value maximization under constraints. In the case of finite investment horizon we use a numerical lattice framework where the constraint is applied and must be satisfied at each lattice node. The implementation details are described in the Appendix. In the following section we discuss the trade-offs that the firm needs to take into consideration when adjusting its investment and optimal default strategies to meet the debt constraints. We then provide numerical results that show the impact of financial constraints and R&D options under different parametizations of the model. The welfare implications of debt financing constraints are discussed in section III. In section IV we endogenize the constraint by providing a model with differential information in volatility or the growth rate between debt holders and equity holders.

II. The effect of financing constraints on the firm

The base case: the model without constraints

In this first subsection we provide results for the unconstrained model (Extended-Leland/MS). We also compare the Extended-Leland/MS model that captures both investment flexibility and the net benefits of debt with Leland's model that captures only the net benefits of debt and McD&S that captures investment timing flexibility only. The comparisons provide insights on the relative importance of investment timing flexibility and the net benefits of debt[4]. Table I provides firm values for the three models and then the (%) net gain that has the following decomposition in the (%) gain of investment flexibility and (%) gain in net benefits of debt:

$$\%\text{ Net Gain} = \frac{F(V) - F^i(V)}{F^i(V)} = \frac{\left[E(V-I) - E^i(V-I)\right]}{F^i(V)} + \frac{\left[NB - NB^i\right]}{F^i(V)} \qquad (11)$$

where i = {McD&S, Leland}. We keep the base case parameter values of Leland (1994) and we use a positive opportunity cost δ of 6%. Other parameters values are as follows:

[4] Leland's model can be obtained by setting $V = V_I$ in equation (9) (no investment timing but optimal capital structure), where for consistency with the other models investment cost I is also subtracted from the firm value of the original Leland model. McD&S model can also be obtained by setting coupons R equal to zero (all-equity firm with an investment option), effectively imposing a zero debt restriction and that the firm never defaults ($V_B = 0$). Furthermore, applying $R = 0$ in equation (9) we get the McD&S investment trigger that equals $V_I = \alpha/(\alpha-1)I$.

value of unlevered assets V =100, risk-free rate r = 0.06, investment cost I =100. For the extended-Leland/MS and the Leland models bankruptcy costs b = 0.5 and tax rate τ = 0.35. The table provides sensitivity analysis for the risk-free rate r, the opportunity cost δ, the volatility of unlevered assets σ, the bankruptcy costs b, and the tax rate τ. When we compare the extended-Leland/MS model with the McD&S, we see that the net gain is due to the net benefits of debt only (at a loss in investment flexibility). When comparing it to the Leland model, the net gain is due to investment flexibility only (at a loss in the net benefits of debt). The relative (%) differences between the extended and the McD&S (Leland) models are at a maximum (minimum) at higher opportunity cost δ, higher risk-free rate r, lower volatility σ, lower bankruptcy costs b, and higher tax rate τ. At low opportunity cost δ, low interest rate r, high volatility σ, low tax rate τ and high bankruptcy costs b the investment timing option is thus more significant . It is thus expected that the effect of financing constraints will be more severe when the net benefits of debt have a more substantial contribution in the extended model value. An interesting observation is on the effect of volatility since it affects the investment flexibility and the net benefits of debt in the opposite direction. An increase in volatility increases the firm value in the McD&S model (investment flexibility increases) but it decreases firm value in the Leland model (net benefits of debt decrease). In the extended-Leland/MS model, those opposite forces result in a non-monotonic function for firm value.

[Insert table I]

Table II shows additional information with respect to the three models. The investment triggers and the bankruptcy triggers are reported first[5]. The other columns show for all models, equity and debt values, optimal coupon and credit spreads, reported at the optimal investment trigger (note that for the standard Leland model, investment takes place immediately at optimal capital structure). We emphasize two observations. First, the investment trigger in the extended model is always lower than in the McD&S model. So, it may seem a priori that the forces benefiting earlier development in the presence of debt (the acceleration of investment benefits and net benefits of debt) dominate. Note, however, that the comparison is for two extreme cases, the extended model at optimal debt, and the McD&S which is effectively a model constrained to zero debt. As we will see in the next section, for in-between cases (with arbitrary levels of debt constraint) this relationship is not monotonic (we observe a U-shape). This means that as debt levels increase the optimal adjustment in the investment trigger may be an increase instead of a decrease. Another important observation relates to debt capacity at different parameter values. Debt levels are higher at higher r, τ and at lower δ and b and have a non-monotonic relation to volatility[6]. Higher debt capacity (at the unconstrained level) would imply large initial adjustments to bring debt to the constrained level.

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[5] Note that in the case of low volatility, $\sigma = 0.05$, we report the theoretical triggers although the current value of V is higher than the investment trigger. The investment option will thus be exercised immediately so that firm value, etc. reported are equal to those of the Leland model, and similarly a new bankruptcy trigger at 66.83 will be relevant.

[6] It is also interesting to note that the bankruptcy triggers in the extended-Leland/MS model are higher than in the Leland model (the optimal coupon is higher in the extended model than in the Leland model) resulting in the same capital structure as in Leland model. For the extended model, we can also see that the bankruptcy trigger behaves non-monotonically with respect to the volatility. As we know from Leland (1994), optimal capital structure (and thus credit spreads) is invariant to the level of unlevered assets and the same holds in the Extended-Leland/MS model.

[Insert table II]

Optimal firm decisions under financing constraints without the R&D option

We start by summarizing the trade-offs involved in meeting the debt constraint (see table III). This will be useful in understanding the impact of constraints and R&D options under different parameters. When the constraint becomes binding the firm should reduce the investment trigger level V_I or the coupon level R (or both) in order to satisfy the constraint. These adjustments involve trade-offs that can be seen through careful inspection of firm value in equation 8b and are summarized as follows. A reduction in the investment trigger provides earlier receipt of investment benefits and of net benefits of debt. On the other hand, a lower investment trigger sacrifices part of the timing flexibility and increases the probability of default thus reducing the expected net tax benefits. A reduction in the coupon level results in a lower default trigger and thus increases the horizon where tax benefits will be received and decreases the expected bankruptcy costs. At the same time, however, it also reduces the level of tax benefits.

[Insert table III]

We now explore the effect of financing constraints on firm and equity value, bankruptcy and investment thresholds, leverage and the credit spread under different model parametizations. In this subsection we assume a perpetual horizon and for the absence of an R&D option. The effect of finite investment horizon and the R&D option are analyzed in the next subsection. In the following figures, firm values are reported at time zero. All other information is for a value of V equal to the optimal investment trigger V_I. Figures 2 and 2a show the implications of financing constraints on firm, the investment and bankruptcy triggers, leverage and the credit spread at different levels of risk-free rate, opportunity cost δ and volatility. The unconstrained case often leads to debt levels above 100% of the required investment capital and to very high firm value and high investment and bankruptcy trigger values. Our constrained borrowing approach should thus be used in most practical applications where debt is at 100% or less of the investment cost. In figure 2 as expected we see that financing constraints decrease firm values. After careful inspection, we see that with a small dividend yield (i.e., for high growth firms) constraints result in a less pronounced (%) decrease in firm value due to the higher importance of investment flexibility at lower δ. The initial adjustment (from unconstrained to constraint levels) for lower δ is more significant since debt capacity levels are high at lower δ. With a small volatility constraints result in a more pronounced (%) decrease in firm value since they are reducing the larger debt finance benefits of low volatility. At lower σ, the initial adjustment to meet the constraint is less significant.

An interesting observation is that debt financing constraints often produce a U-shape in the investment trigger. In our case, the observed U-shape exists because when the constraints start to become binding (at high debt levels), the firm will invest earlier (at lower investment trigger) since as we show in the appendix decreasing this trigger decreases debt value. This permits satisfaction of the constraint in a way that also allows the firm to retain a coupon level as high as possible and thus reduce the loss on the tax benefits of debt. Tax benefits are thus retained at high value and are also obtained earlier.

[Insert figures 2 and 2a]

With stricter level of constraints the firm will place less emphasis on tax benefits and will delay investment in order to enhance the option value. Furthermore, the higher investment trigger in combination with a lower default trigger (due to a lower coupon), reduce the probability of default maintaining any value arising from tax benefits. These latter effects prevail at low levels of debt thus generating the observed U-shape on investment trigger. This result differs from Boyle and Guthrie (2003) since their emphasis is on constraints on cash balances while we focus on constraints on debt[7].

In figure 2a we see that bankruptcy trigger and leverage ratios are decreasing. The fact that lines on the figures may cross shows that some firms may seem to have lower leverage ratios than others even though their optimal (unconstrained) leverage ratios would have been higher. The last part of that figure shows the impact of constraints on credit spreads, which is non-linear. Compared to the base case, lower δ results in lower credit spreads. This reflects lower bankruptcy risk since, as shown in figure 2, the investment trigger is higher, the bankruptcy trigger is lower, and the (risk-neutral) drift is higher. With stricter constraints, the difference between the levels of the bankruptcy and the investment triggers is larger, thus the credit spreads are further reduced. Again compared to the base case, for lower interest rates credit spreads are higher. This now reflects higher bankruptcy risk, since although both the investment and the bankruptcy trigger are somewhat lower, the (risk-neutral) drift is lower. With stricter

[7] The trade-offs in their model is that an increase in cash balances makes early investment more attractive but also reduces the risk that the constraint becomes binding in the future thus also enhancing the option value of delaying investment.

constraints, the investment trigger goes up and the bankruptcy trigger goes down thus further decreasing bankruptcy risk and credit spreads. The case of volatility is more complex. Lower volatility reduces the gap between the two triggers, which would increase bankruptcy risk, but with lower volatility the probability of hitting the bankruptcy trigger may be reduced and apparently this latter effect may become (as in this case) more important.

In figures 3 and 3a we similarly see the implications of financing constraints on firm, the investment and bankruptcy triggers, leverage and the credit spread at different levels of bankruptcy costs and tax rates[8]. We observe that for low tax rates, stricter constraints have a small effect on firm value and the investment trigger since for low tax rates the net benefits of debt are low (the firm has already set the investment trigger so that it optimizes the option on unlevered assets). In figure 3a we see that leverage and more importantly credit spreads tend to converge in the constrained region (whereas in the unconstrained region there can be significant differences for different levels of bankruptcy costs and tax rates). In the constrained region the optimal bankruptcy trigger for low tax rates may be higher than in the base case. Reduced bankruptcy costs have a smaller effect on firm value, default trigger, leverage and the credit spreads at the constrained region.

[Insert figure 3 and 3a]

Effect of R&D options and finite investment horizon

[8] To the left of figure 5, all values for zero debt converge to the same point which corresponds to the McD&S case, since the bankruptcy costs and tax rates affect the net benefits of debt only.

The earlier analysis was under the assumption of perpetual investment horizon and no R&D option. Table IV shows firm values in the extended-Leland/MS model with sensitivity to the investment option maturity, the level of debt constraints and in the presence of early-stage R&D options. We assume that the cost of exercising the R&D option is zero. Effectively, the R&D option can be exercised if its cost is less than the increase in value relative to the base (no R&D option) case. Note that with very high maturities (T = 50) the numerical solution approximates the analytic model (see base case of table II). Reduced option maturity results in a decreased firm value as expected. This result appears in both constrained and unconstrained case, and both in the presence and in the absence of R&D growth options. Koussis, Martzoukos and Trigeorgis (2007) have shown that firm value is increasing in both the mean impact and volatility of growth options for an all-equity firm. With debt, we show here that they also increase firm value despite a potentially negative effect on the net benefits of debt. An interesting observation is that in the presence of R&D options, the effect of constraints at lower maturities is lessened.

[Insert table IV]

We emphasize that our assumption of R&D options that when exercised have a random outcome differs from the growth option component of Childs, Mauer and Ott (2005) and Mauer and Ott (2000). In our case, exercise of the (equity financed) pre-investment R&D growth option affects the distribution of project value instantaneously (an impulse-control) and uncertainty reverts to “normal” just afterwards. This situation is particularly relevant for risky start-up ventures involving initial technical uncertainties. We do not investigate issues of “asset substitution”, i.e., equity holders engaging in riskier strategies ex-post to debt agreement thus transferring wealth from bond holders to equity holders (like for example in Leland, 1998).

In tables V and VI that follow we investigate the impact of R&D options in further detail with the assumption of a perpetual investment horizon. All the values reported are the expected ones due to the presence of R&D uncertainty since we report them conditional on the exercise of the R&D. Table V shows numerical results for the effect of R&D on firm value and its two components, the expected value of unlevered assets and the expected net benefits of debt. In the same table we explore the effect of exercise of the R&D option in the presence of financing constraints on debt. Concentrating on the first panel (the case with no constraints) we see that in all models firm values are increasing in both the volatility of R&D and its expected impact. This is in contrast to the effect of an increase in the Brownian volatility (see discussion in table II) that decreases firm value in the Leland model (and creates a non-monotonic shape in the extended model). An increase in volatility increases the option on unlevered assets but may decrease the net benefits of debt. In both the extended Leland and the Leland model, an increase in the mean impact has a positive effect on both the option on unlevered asset and the net benefits of debt.

[Insert table V and VI]

The second and third panel of table V show the effect of different levels of financing constraints on firm value and its components. For a given debt constraint the effect of

R&D is like before. Comparing the panels with increasingly strict debt constraints we still see (as expected) a decrease in firm values. The driver of the decrease in firm value is mostly due to the decrease in the net benefits of debt while we do not necessarily observe a decrease in expected option on unlevered assets. This is because of the often observed U-shape of the investment trigger (see discussion on figure 2) where the firm adjusts its investment policy to stricter constraints.

Table VI presents more information for the expected optimal capital structure (expected leverage) and the expected credit spread. Note that firm values (see table IV) are equal to expected equity plus expected debt minus the expected investment cost. We see that (in both the unconstrained and the constrained cases) expected equity is increasing in both R&D volatility and its mean impact in the extended model while in Leland's model it is only increasing in the mean impact (but may be decreasing in growth option volatility). In the unconstrained case expected leverage and expected credit spreads stay unchanged and expected debt is affected positively in the impact and volatility of the R&D. With the simultaneous presence of R&D and stricter debt constraints we see a decrease in expected optimal leverage and an accompanying decrease in expected credit spreads. This is to be contrasted with the case of an increase in Brownian volatility that would increase credit spreads. In this case the volatility acts favourably since information gets revealed before investment and does not affect uncertainty afterwards. We also see that in both the unconstrained and the constrained cases an increase in R&D volatility has an ambiguous effect on the expected cost[9]. An

[9] Note that expected costs reflect the probability of development. Sarkar (2000) also shows in a real options setting that an increase in volatility may speed up instead of delay investment.

increase in the mean impact of the growth option increases expected cost (since it increases the probability of development).

III. Taxes and welfare effects of debt financing constraints

Our earlier analysis can be extended in order to draw some insights on the effect of debt financing constraints on government taxes and social welfare. In this section we model the firm's revenues as the underlying stochastic variable since taxes are contingent on the continuous flow of revenues that are generated by the firm. We use P to denote the continuous yearly net revenues before taxes. The following relationship keeps consistency with our earlier analysis:

$$V = \frac{P}{\delta}(1-\tau) \tag{12}$$

Effectively, the value of unlevered assets is the present value of after tax income stream (we set operational costs to zero and we exclude the option to abandon that where used in the Mauer and Sarkar (2005) model). So, we use $P = \frac{V}{(1-\tau)}\delta$, $P_B = -\frac{\beta}{(1-\beta)}\delta\frac{R}{r}$ and $P_I = \frac{V_I\,\delta}{(1-\tau)}$. Following Mauer and Sarker (2005)[10] government taxes at the investment threshold can then be defined as:

$$T(P_I) = \frac{\tau(P_I - R)}{r} - \frac{\tau(P_B - R)}{r}\left(\frac{P_I}{P_B}\right)^{\beta} + \frac{\tau P_B}{r}\left(\frac{P_I}{P_B}\right)^{\beta} \tag{13}$$

[10] Although the definition of taxes and social welfare function are the same as in Mauer and Sarkar (2005) in our case their level are determined at the constraint level though the optimal reaction of the firm under constraints.

The first term reflects the perpetual flow of government taxes received at the time of investment which are adjusted by the taxes foregone at default (second term). The last term reflects the taxes received from the unlevered firm which remains after default. Obviously, government revenues may be directly increased the lower the debt level used by the firm, since the firm's revenues after coupon reductions are higher. However, since the government does not control the firm's optimal investment and default adjustments under constraints, government taxes may increase at a decreasing rate or even decrease under constraints. For example, if constraints cause earlier investment by the firm this may reduce the level of revenues and thus the taxes generated by the firm. Since these adjustments are not linear and involve adjustments in many dimensions, it is very difficult to know a priori what the effect of constraints on government taxes would be. Social welfare value at time zero is calculated as the sum of representative firm and government taxes[11]:

$$SW = \left(F(P_I) + T(P_I)\right)\left(\frac{P}{P_I}\right)^{\alpha} \tag{14}$$

Note that the function of firm value *F*(.) is the same like in equation (6) evaluated with respect to the revenue level. As we have shown in earlier sections, firm value is decreasing in the level of financing constraints. Since taxes may be increasing the

[11] By construction this is a partial equilibrium analysis. Thus this model does not endogenize equilibrium economy credit levels and monetary policy implications (see for example discussions in Bernanke and Gertler, 1995). Monetary policy interventions may affect the cost of external (debt) financing (the balance sheet effect) or directly limit the available credit in the economy through the bank lending channel. The availability of credit for firms will ultimately be determined after market frictions (e.g. due to asymmetric information and moral hazard) take place. We thus note that the maximum level of social welfare may not be achieved since by definition the level of constraint is exogenous and is due to other factors that are beyond government control.

maximum level of social welfare may be determined at a constraint level as will be the case in the numerical investigations that we perform below.

In figure 4 we see the effects of financing constraints on welfare and its components (firm value and taxes) for the base case parameters used in the previous section. Figure 4a shows the results for a lower volatility rate and figure 4b for a lower tax rate (τ).

[Insert figure 4, 4a, and 4b]

Using the base case parameters we find government taxes increase the lower the debt level used. The opposite direction of firm value and government taxes creates a social welfare optimum at a constrained level of around 50% of total investment cost. We also note that government taxes increase at a decreasing rate for this set of parameters. Government taxes are driven by the complex behaviour of the optimal investment and default trigger of the firm under constraints[12].

For a lower volatility level of 15% (see figure 4a) social welfare is maximized at a higher level of debt of 75%. Taxes are increasing at a high rate initially (as the constraint starts to become binding), but for very low level of debt taxes remain relatively flat. For a lower tax rate of 15% (see figure 4b) social welfare is maximized at lower debt levels (at about 25% of investment). Since the firm has fewer benefits to obtain from tax credits at lower debt levels, its value is relatively flat (although decreasing) at lower debt levels. Taxes are also relatively flat but they are shown to increase at relatively higher rate at stricter constraints thus driving the observed result.

[12] Remember that the investment trigger exhibits a U-shape and the default trigger is higher the more debt is used.

IV. Endogenous debt constraints due to differential information between equity and debt holders

Up to now we have assumed exogenous constraints. Financing constraints though, can be caused endogenously by differential beliefs on the true estimates of volatility or the growth rate (determined by the opportunity cost rate δ). We assume that each party truthfully communicates its beliefs to the other. Next we describe how we model differential information in volatility. Similar analysis applies for the growth rate. Numerical results are presented for both cases.

Equity holders will use their own estimate to optimize the bankruptcy decision. The default trigger determined using their estimate of volatility (that affects V_B through the auxiliary parameter $\beta(e)$ is:

$$V_B(e) = \frac{-\beta(e)}{(1-\beta(e))}\frac{R(1-\tau)}{r}$$

$$\beta(e) = \frac{1}{2} - \frac{(r-\delta)}{\sigma_e^2} - \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma_e^2}\right)^2 + \frac{2r}{\sigma_e^2}} < 0 \tag{15}$$

where σ_e^2 is the estimate of volatility perceived by equity holders. Equity value is then given by:

$$E(V_I) = V_I - \frac{R}{r} + \tau\frac{R}{r} + \left[\left[\frac{R}{r}\right] - V_B(e) - \left[\tau\frac{R}{r}\right]\right]\left[\frac{V_I}{V_B(e)}\right]^{\beta(e)} \tag{16}$$

For a given investment trigger, debt holders will decide on the amount of debt to be given based on their estimate of volatility. Debt holders will determine the amount of debt by:

$$D(V_I) = \frac{R}{r} + ((1-b)V_B(e) - \frac{R}{r})(\frac{V_I}{V_B(e)})^{\beta(d)} \tag{17}$$

Note that debt holders use their own perception of the volatility that affects parameter $\beta(d)$ and in turn their perceived probability of default and the expected present value of debt.

Equity holders working backwards will take into consideration debt holders valuation when they decide about the optimal timing of investment which is found by maximizing firm value:

$$F(V) = \left[E(V_I) + D(V_I) - I\right]\left(\frac{V}{V_I}\right)^a \tag{18}$$

where

$$a = \frac{1}{2} - \frac{(r-\delta)}{\sigma_e^2} + \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma_e^2}\right)^2 + \frac{2r}{\sigma_e^2}} > 1$$

Note that $D(V_I)$ is the value of debt as perceived by debt holders. The optimal investment trigger is then found by solving the following first order condition:

$$1+\beta(e)\left((1-\tau)\frac{R}{r}-V_B(e)\right)\left(\frac{V_I}{V_B(e)}\right)^{\beta(e)}\left(\frac{1}{V_I}\right)+\left(\beta(d)((1-b)V_B(e)-\frac{R}{r}\right)\left(\frac{V_I}{V_B}\right)^{\beta(d)}\left(\frac{1}{V_I}\right)$$
$$-\alpha\left(\frac{1}{V_I}\right)\left(\left(V_I-(1-\tau)\frac{R}{r}+(1-\tau)\frac{R}{r}-V_B(e)\right)\left(\frac{V_I}{V_B(e)}\right)^{\beta(e)}+\frac{R}{r}+\left((1-b)V_B(e)-\frac{R}{r}\right)\left(\frac{V_I}{V_B(e)}\right)^{\beta(e)}-I\right)=0 \tag{19}$$

Equation 19 includes debt holders' differential beliefs about the volatility since the debt value incorporates debt holders estimate. Similar analysis can be applied for differential perceptions about the dividend yield (affecting the perceived growth of unlevered assets).

Table VII(a) presents numerical results with varying degree of differential information in terms of volatility between the two stake holders. The upper panel of the first table shows results when debt holders believe that actual volatility is lower than that perceived by equity holders. In this case equity holders will invest earlier than in the symmetric case because they can use higher leverage. Equity holders also default at higher default trigger compared to the symmetric case. Note that in this case firm value increases substantially since equity holders can acquire cheap debt. In the more interesting case where debt holders believe that volatility is higher, equity holders will delay investment and also default at a later point. This enhances the value of equity and reduces debt and firm value. This unfavourable for the equity holders differential information effectively acts as a binding constraint on debt since we observe that debt levels and optimal leverage ratios are lower than in the symmetric case. Credit spreads seem to be lower than in the symmetric case when debt holders perceive lower volatility than equity holders and this seems to be reversed when their perception is higher. For very high (unfavorable) asymmetry levels credit spreads get lower than the symmetric case (because of the extremely low debt levels used).

Table VII(b) shows results for the case of differential information in terms of growth rate estimates. A higher level of perceived δ implies a lower perceived level of growth. Our results are similar to the case of differential information about volatility including the behavior of the credit spreads. Importantly, when debt holders perceive lower growth rate of the assets the optimal investment trigger is higher, the optimal default trigger is lower and debt levels and leverage ratios fall. Effectively, lower perceived growth rates by debt holders act as a constraint on the level of debt used.

The differential information cases analyzed create some results that differ from the exogenously imposed constraint analyzed in the previous section. For example, one important difference is that we no longer observe a U-shape in the investment trigger. In the case of unfavorable differential information we now observe that equity holders will optimally delay investment. Our analysis adds to the literature analyzing the underinvestment problem (see for example, Moyen, 2002 and Mauer and Ott, 2000). In this literature equity holders decision to delay investment (and thus underinvest) exists when there is existing debt and new investments are financed solely with new equity. Equity holders underinvest since the new investment creates shared benefits with existing debt holders (while equity holders alone bear extra risk. Leland (1998) and Mauer and Sarkar (2005) discuss overinvestment incentives by equity holders. In Leland (1998) the overinvestment exists because of asset substitution, i.e., equity holders invest in riskier project ex post to agreed debt levels. Similarly, in Mauer and Sarkar (2005) equity holders maximize the value of equity instead of total (levered) firm value. Our model provides an alternative explanation based on differential beliefs about the volatility of assets or growth that may justify over or under investment. In the more interesting case that we have analyzed, debt holders have beliefs of higher volatility or lower growth of assets that cause equity holders to underinvest (delay investment) as a way to mitigate the problem of unfavourably priced debt.

IV. Summary

In this paper we have studied the effect of capital constraints on the firm's optimal investment and bankruptcy policy, optimal leverage and credit spreads. Our model provides insights that may also be important for empirical research. We show that financing constraints have a more significant relative impact on firm values at higher levels of competitive erosion, riskless rate of interest and taxes, and lower volatility and bankruptcy costs. Financing constraints also reduce leverage and credit spreads in a nonlinear fashion. An important observation is an often observed U-shape of the investment trigger as a function of the constraint. This result is driven by the trade-off between investment timing flexibility and the net benefits of debt. Exercise of pre-investment R&D growth options increase firm value, although they may decrease the expected net benefits of debt. In the presence of R&D growth options, the impact of debt financing constraints at lower maturities is more significantly reduced than longer maturity options. The firm's optimal investment and default decisions under constraints have implications for the taxes raised by the government. The trade-off between firm decrease and government taxes increase at stricter debt constraints may drive a social optimum at a constraint level of debt.

In the final part of the paper we endogenize debt constraints by considering differential beliefs between debt and equity holders with respect to the volatility or the growth of assets. We show that when debt holders perceived estimate of volatility of assets are higher or when their perceived estimate of the growth rate of assets is lower, equity holders have to reduce optimal leverage and delay investment.

Appendix

Solution to the constrained optimization problem

Starting with the unconstrained optimization problem we create a dense grid of equally spaced coupon values and for each coupon value we find the optimal investment trigger by solving the first-order condition for the investment trigger (see equation (9)) using a standard bisection method[13] (see for example Judd, 1998). The locus of the solutions is depicted by the upper bold curve in figure A1 for the base case parameter values. We then optimize with respect to coupon by selecting the combination that gives the maximum firm value from the created locus (optimal value denoted with the upper right rhomb in figure A1). We verify that each point on the locus represents a global optimum (for each coupon value) by performing a dense grid search for different levels of the investment trigger (above and below the optimal).

The constrained problem is defined in equation (10) of the main text and involves a selection of the optimal coupon (optimal capital structure) and the investment trigger. The condition for the default trigger (see equation (7)) should also always be satisfied. For each coupon value we select the optimal investment trigger by additionally ensuring that the constraint is satisfied. We use the previous approach as long as the constraint is not binding ($D(V_I) < D^{\max}$) and when the constraint becomes binding we reduce the

[13] The grid covers a large range of coupon values with maximum values reaching R = 1,000 effectively tracing through the firm value function.

investment trigger so as to exactly meet the constraint by satisfying $D(V_I) = D^{\max}$. This point is unique since debt is an increasing function of the investment trigger:

$$\frac{\partial D(V_I)}{\partial V_I} = \beta \left(\frac{V_I}{V_B} \right)^{\beta - 1} \frac{R}{r} \left[\frac{-\beta}{(1-\beta)} (1-\tau)(1-b) - 1 \right] > 0 \qquad \text{(A1)}$$

For r>0 , $0 \le \tau < 1$ and $0 \le b \le 1$ the above inequality is ensured from β<0 and $\frac{-\beta}{(1-\beta)} < 1$. We then select the maximum firm value generated by this locus of solutions. Our algorithm produces a set of solutions that is depicted for illustration in figure A1. The curves below the bold curve show the locus of solutions for various levels of constraints. Starting from the top we have the unconstrained case and the cases where maximum debt equals 100, 75, 50 and 25. The constrained lines overlap with the unconstrained as long as the constraint is not binding (towards the left). For each case a rhomb identifies the point of maximum firm value. The optimal solutions for the constrained problems are usually located at or near the unconstrained curve. The case of zero debt (i.e., the McD&S model) is reflected by the upper left rhomb. Again we verify that each point on the locus represents a global optimum (for each coupon value) by performing a dense grid search for lower level of the investment trigger.

[Insert figure A1]

Finite horizon of investment option

The finite investment option horizon is implemented using a numerical binomial lattice scheme. With N lattice steps we have the up and down lattice moves and the probabilities of up and down equal to:

$$
\begin{aligned}
&u = \exp(\sigma\sqrt{\frac{T}{N}}), \qquad d = 1/u \\
&p_u = \frac{\exp\left((r-\delta)T\right) - d}{u-d}, \quad p_d = 1 - p_u
\end{aligned}
\tag{A2}
$$

For optimal coupon selection at each value of V we apply the condition $\frac{\partial V^L(V)}{\partial R} = 0$ which gives:

$$
\frac{\tau}{r}\left(1-\left(\frac{V}{V_B}\right)^{\beta} + \beta\frac{1}{r}\left(\frac{V}{V_B}\right)^{\beta}\right) + b\frac{\beta}{(1-\beta)}\frac{(1-\tau)}{r}\left(\frac{V}{V_B}\right)^{\beta} + \beta b V_B\left(\frac{V}{V_B}\right)^{\beta}\frac{1}{R} = 0 \tag{A3}
$$

with V_B given in equation (7) of the main text. We apply equation (A3) at *each* node of the lattice and we additionally allow for the early exercise of the investment option. At exercise, option value at each node equals $E(V) + D(V) - I$ where we use the analytic values of $E(V)$ and $D(V)$ given by equations (4) and (5) of the main text at $V_I = V$. For the constrained problem and for each value of V we again apply a grid search at various coupon levels to find the constrained optimal.

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Figure 1: Extended-Leland/MS model with R&D growth option, investment option, and debt financing constraints

Time 0: **R&D growth option decision ($F^*(V)$):**

- Exercise of R&D options, or
- exercise investment option, or

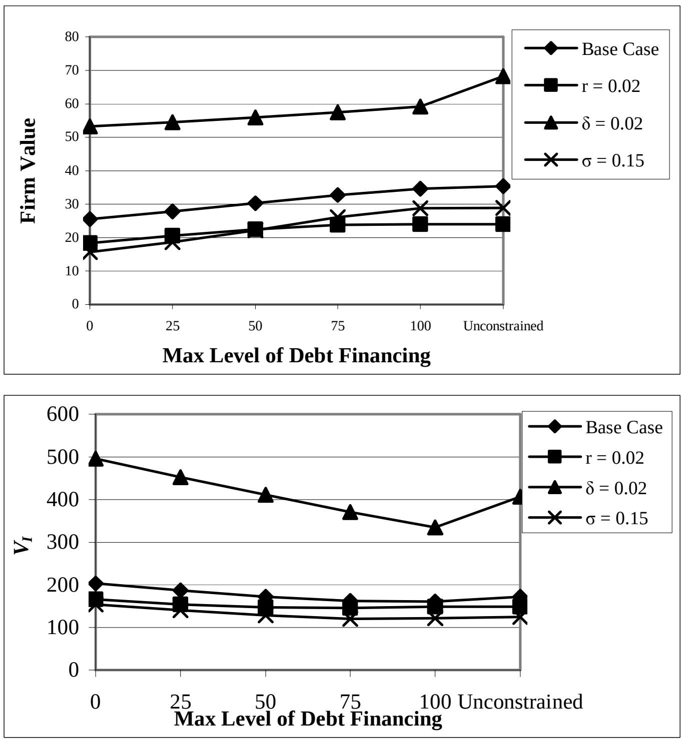
Time $t \in [0, T]$ (T is infinite in the analytic solution case): **Investment and capital structure decision ($F(V)$):**

- Wait, or
- exercise investment option at t_I when V hits optimal investment trigger V_I; determine optimal coupon subject to financing constraints, and optimal default trigger V_B

Time $t > t_I$ until ∞: **Default decision ($E(V)$):**

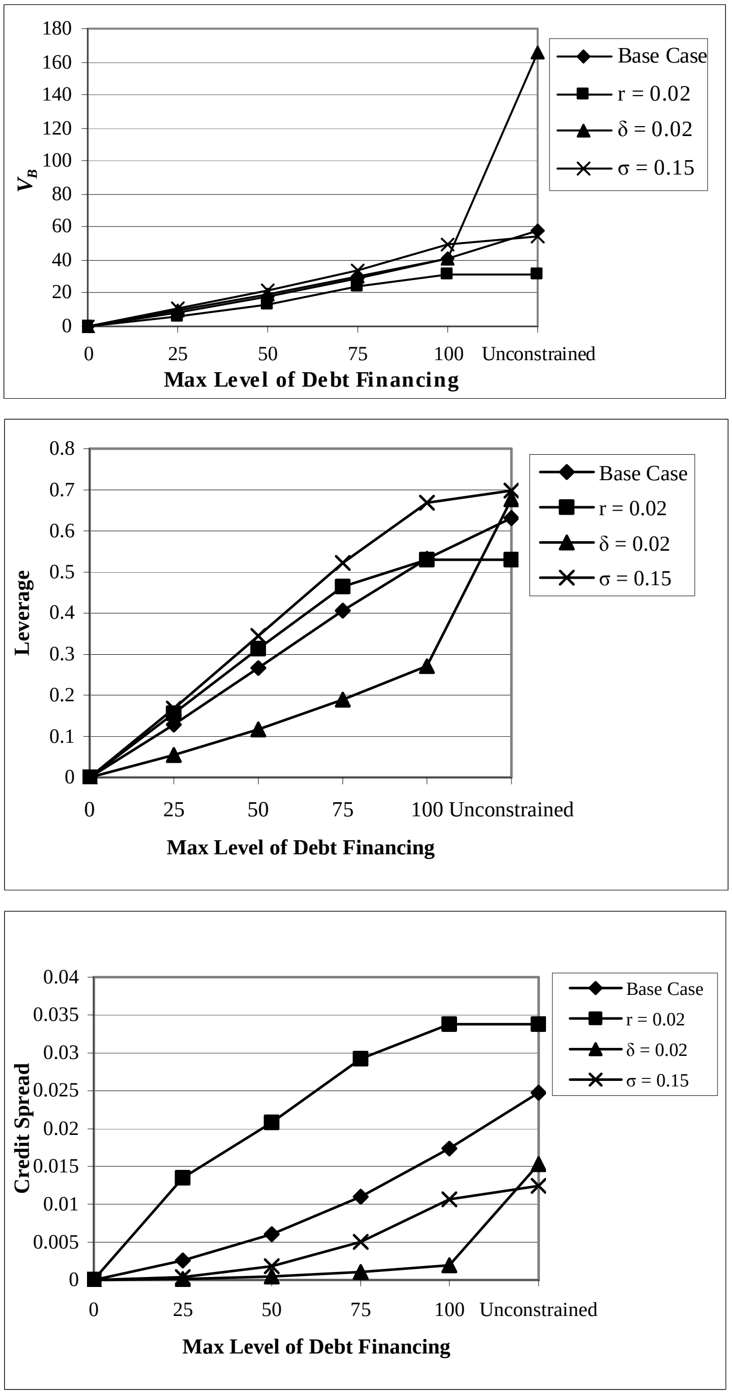
- Default if $V \leq V_B$

Figure 2: Firm value, equity values, and investment trigger as a function of maximum levels of debt: Sensitivity with respect to *r*, *δ* and *σ*.



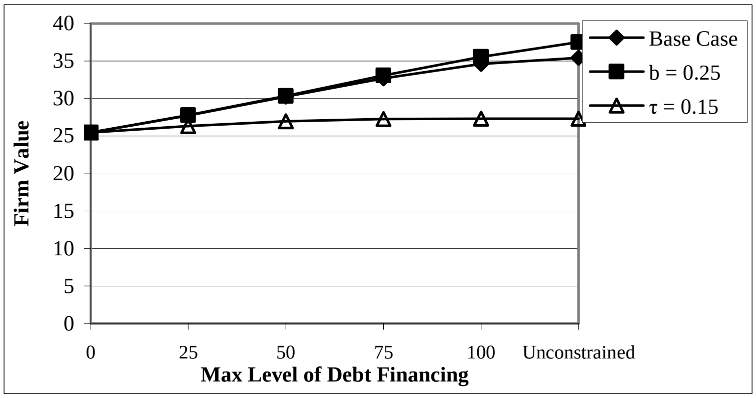
Notes: Base case used: Value of unlevered assets V =100, risk-free rate r =0.06, opportunity cost δ = 0.06, investment cost I =100, volatility of unlevered assets σ = 0.25, tax rate τ = 0.35 and bankruptcy costs b= 0.5. Sensitivity with respect to the risk free rate r, opportunity cost δ, and volatility σ.

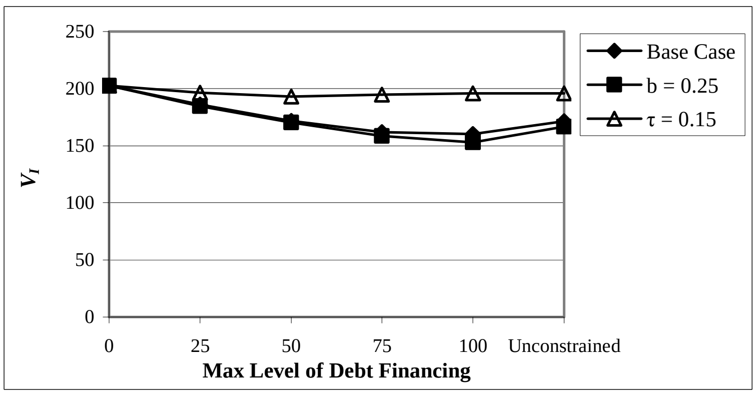
Figure 2a: Bankruptcy trigger, leverage and credit spreads as a function of maximum levels of debt: Sensitivity with respect to *r*, *δ* and *σ*.



Notes: Base case used: Value of unlevered assets V =100, risk-free rate r =0.06, opportunity cost δ = 0.06, investment cost I =100, volatility of unlevered assets σ = 0.25, tax rate τ = 0.35 and bankruptcy costs b= 0.5. Sensitivity with respect to the risk free rate r, opportunity cost δ, and volatility σ.

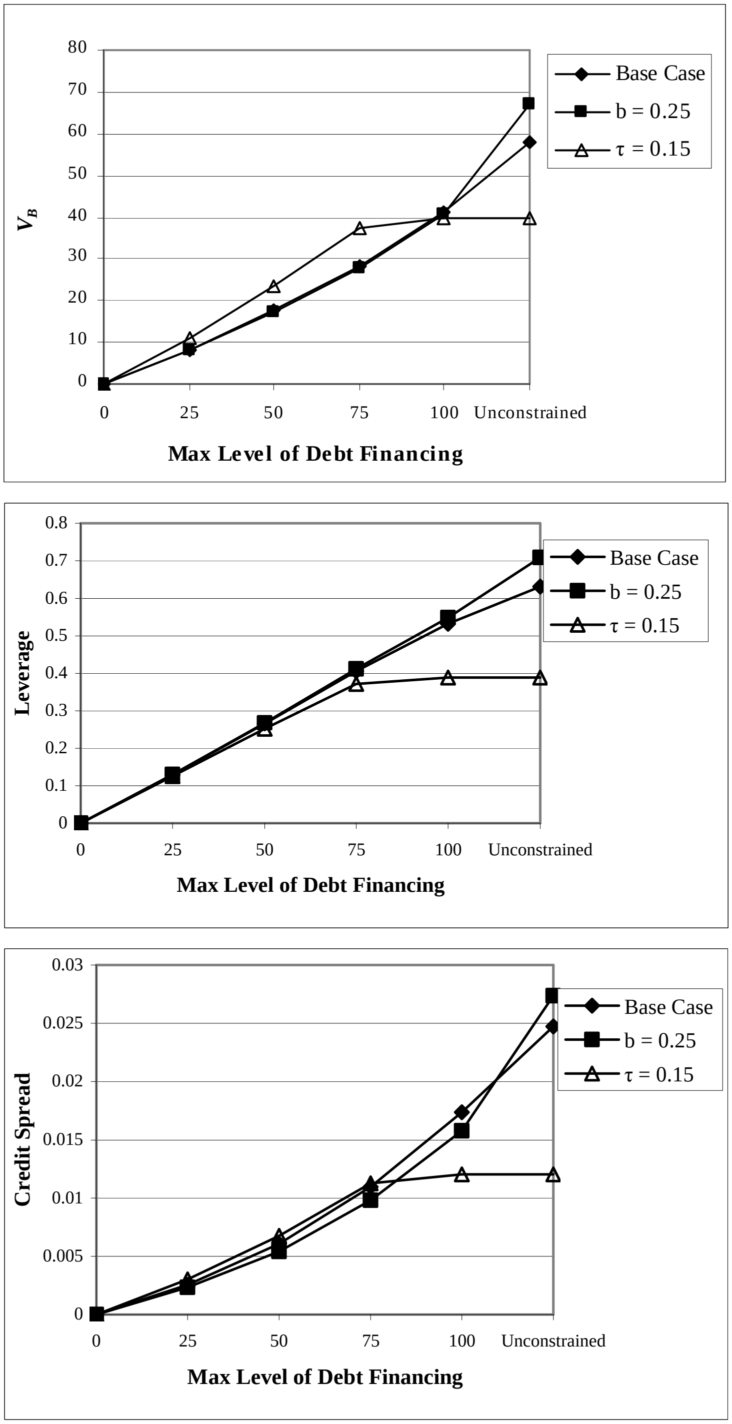
Figure 3: Firm value, equity values, and investment trigger as a function of maximum levels of debt: Sensitivity with respect to τ and b.





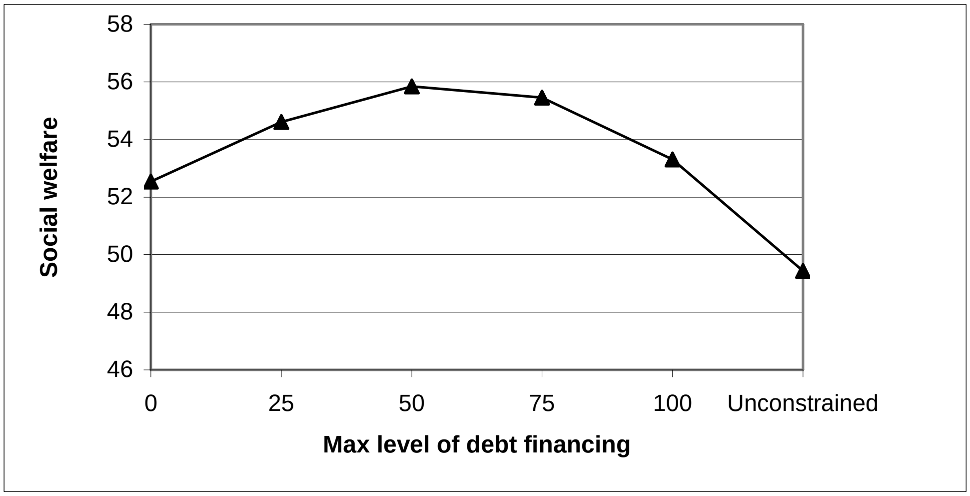
Notes: Base case parameters used: Value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ = 0.06, investment cost I=100, volatility of unlevered assets σ = 0.25, tax rate τ = 0.35 and bankruptcy costs b=0.5. Sensitivity with respect to bankruptcy cost b and tax rate

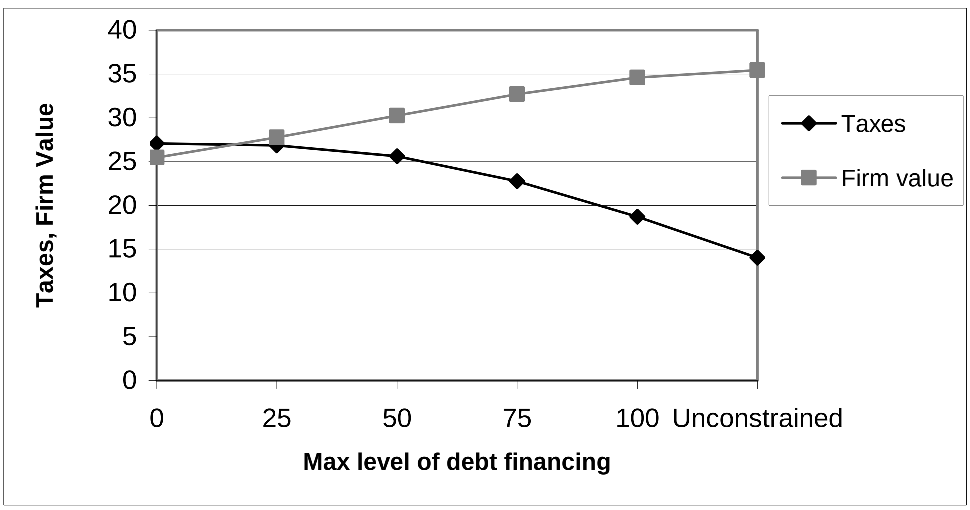
Figure 3a: Bankruptcy trigger, leverage and the credit spread as a function of maximum levels of debt: Sensitivity with respect to *τ* and *b*.



Notes: Base case parameters used: Value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ = 0.06, investment cost I =100, volatility of unlevered assets σ = 0.25, tax rate τ = 0.35 and bankruptcy costs b=0.5. Sensitivity with respect to bankruptcy cost b and tax rate τ.

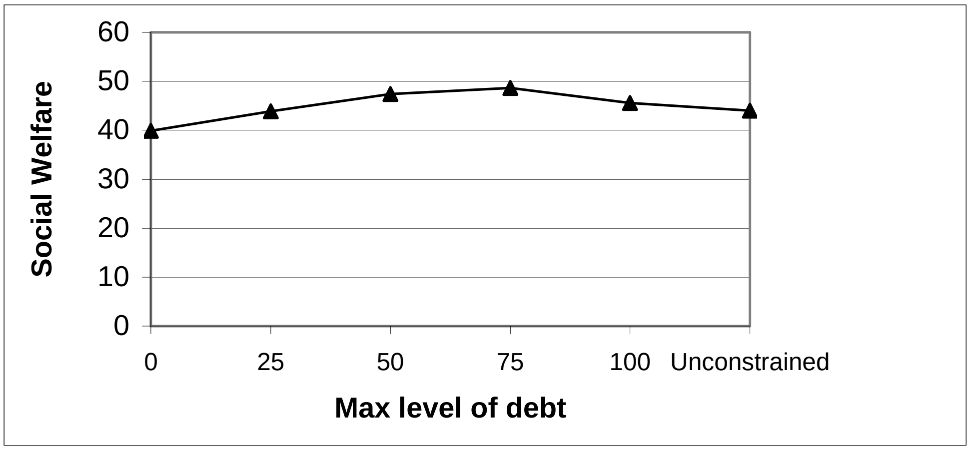
Figure 4: Social Welfare and its components, firm value and taxes as a function of debt financing constraints

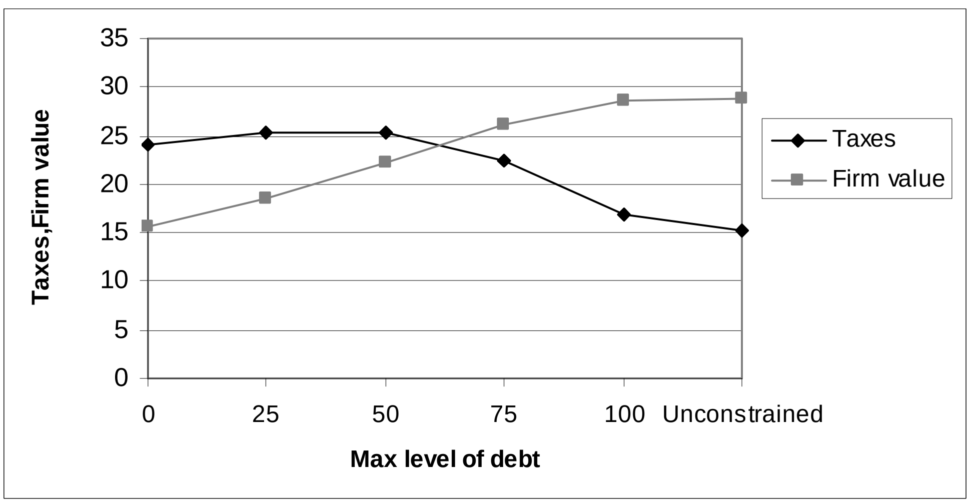




Notes: Base case used: product revenues P = 9.231 which is equivalent to a value of unlevered assets V =100. Risk-free rate r =0.06, opportunity cost δ = 0.06, investment cost I =100, volatility of unlevered assets σ = 0.25, tax rate τ = 0.35 and bankruptcy costs b= 0.5.

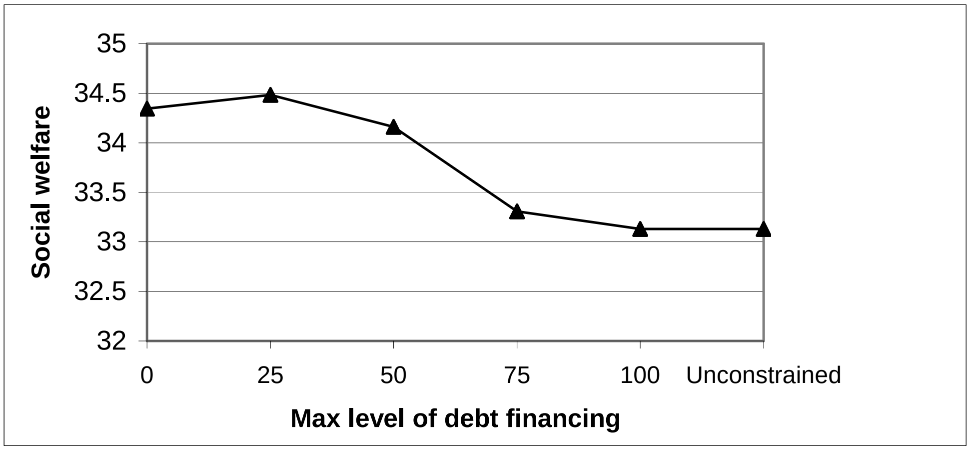
Figure 4a: Social Welfare and its components, firm value and taxes as a function of debt financing constraints: Lower volatility (σ = 0.15)

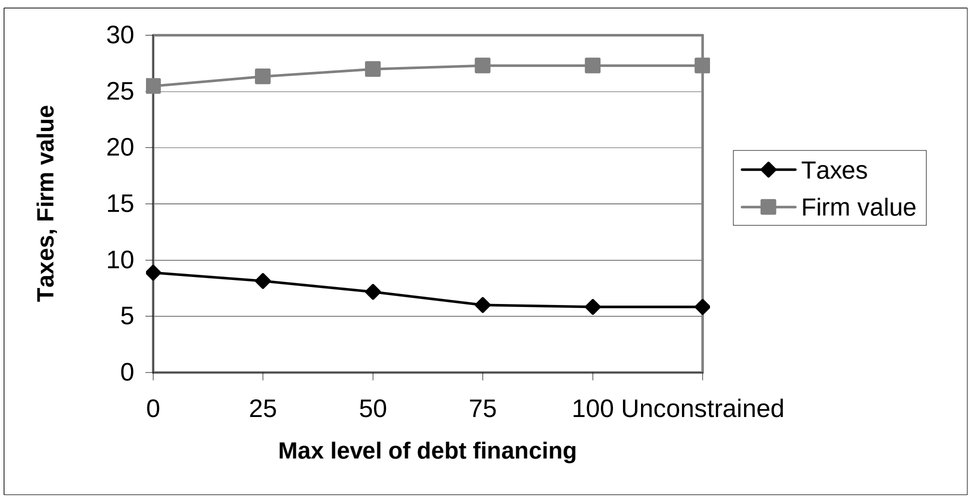




Notes: Base case used: product revenues $P = 3.077$ which is equivalent to a value of unlevered assets V =100. Risk-free rate r =0.06, opportunity cost δ = 0.06, investment cost I =100, volatility of unlevered assets $\sigma = 0.15$, tax rate $\tau = 0.35$ and bankruptcy costs b= 0.5.

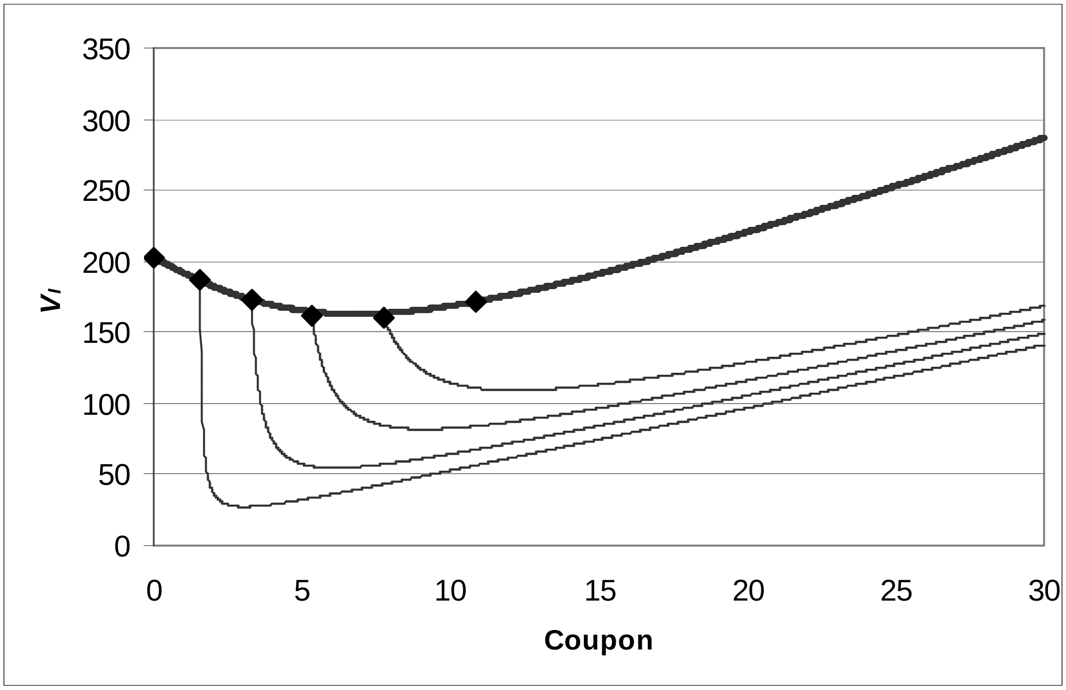
Figure 4b: Social Welfare and its components, firm value and taxes as a function of debt financing constraints: Lower tax rate (τ = 0.15)





Notes: Base case used: product revenues P = 7.059 which is equivalent to a value of unlevered assets V =100. Risk-free rate r =0.06, opportunity cost δ = 0.06, investment cost I =100, volatility of unlevered assets σ = 0.25, tax rate τ = 0.15 and bankruptcy costs b= 0.5.

Figure A1: Illustration of the constrained optimization solution



Notes: The bold line represents the unconstrained solutions, and the lines below constraints at a level of debt (starting from top) equal to 100, 75, 50, and 25. Rhombs represent the unconstrained and constrained optima. For the base case we use a value of unlevered assets V = 100, risk-free rate r = 0.06, oopportunity cost δ = 0.06, investment cost I = 100, volatility of unlevered assets σ = 0.25, tax rate τ = 0.35 and bankruptcy costs b= 0.5.

Table I: Comparison of three models with various levels of flexibility - firm value and investment and debt financing gains analysis

	Firm Value			Ext.-Leland/MS vs McD&S			Ext.-Leland/MS vs Leland		
	Ext.-Leland/MS	McD&S	Leland	% Gain $E[V\text{-}I]$	% Gain *NB*	% Net Gain	% Gain $E[V\text{-}I]$	% Gain *NB*	% Net Gain
Base	35.42	25.48	18.18	-0.03	0.42	0.39	1.36	-0.41	0.95
$r = 0.02$	23.92	18.28	11.19	-0.03	0.33	0.31	1.59	-0.46	1.14
$r = 0.04$	29.48	21.74	14.73	-0.03	0.39	0.36	1.43	-0.43	1.00
$r = 0.08$	41.38	29.27	21.34	-0.03	0.45	0.41	1.33	-0.39	0.94
$\delta = 0.02$	68.30	53.27	21.95	-0.01	0.29	0.28	2.41	-0.30	2.11
$\delta = 0.04$	47.29	35.49	19.96	-0.02	0.35	0.33	1.75	-0.38	1.37
$\delta = 0.08$	28.05	19.28	16.68	-0.05	0.51	0.45	1.10	-0.42	0.68
$\sigma = 0.05$	35.99	5.30	35.99	-1.00	6.79	5.79	0.00	0.00	0.00
$\sigma = 0.15$	28.88	15.69	23.76	-0.17	1.01	0.84	0.55	-0.33	0.22
$\sigma = 0.35$	43.09	34.40	15.04	-0.01	0.26	0.25	2.26	-0.40	1.87
$b = 0.05$	39.93	25.48	25.58	-0.06	0.63	0.57	0.93	-0.37	0.56
$b = 0.25$	37.51	25.48	21.67	-0.04	0.52	0.47	1.12	-0.39	0.73
$b = 0.75$	33.94	25.48	15.65	-0.02	0.36	0.33	1.59	-0.42	1.17
$\tau = 0.15$	27.30	25.48	3.57	0.00	0.07	0.07	7.12	-0.48	6.64
$\tau = 0.25$	30.41	25.48	9.38	-0.01	0.20	0.19	2.69	-0.45	2.24
$\tau = 0.45$	43.43	25.48	31.04	-0.09	0.80	0.70	0.75	-0.35	0.40

Notes: "Ext.-Leland/MS" refers to the main model used with investment and debt financing gains. "McD&S" refers to McDonald and Siegel (1986) model of the perpetual investment option and "Leland" to the Leland (1994) model with optimal debt financing and no investment flexibility. Base case used for all models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ = 0.06, volatility σ = 0.25, investment cost I = 100. For the Ext.-Leland/MS and the Leland model we use bankruptcy costs b = 0.5, tax rate τ = 0.35. The notation "% Gain $E(V\text{-}I)$" refers to the % change in value of the option on unlevered assets and "% Gain *NB*" refers to the % change in the net benefits of debt relative to the other two models. Sensitivity analysis is with respect to the risk-free rate r, opportunity cost δ, volatility of unlevered assets σ, bankruptcy costs b, and the tax rate τ.

Table II: Comparison of three alternative with various levels of flexibility - Investment and bankruptcy triggers, optimal leverage, optimal coupons and credit spreads

	Inv. Trigger (V_I)		Bankr. Trigger (V_B)		Optimal Capital Structure at Investment Trigger V_I: Equity		Debt		Optimal Leverage		Optimal Coupon		Credit Spread	
	Ext. – Leland/MS	McD&S	Ext.- Leland/MS	Leland	Ext.- Leland/MS	Leland	Ext.- Leland/MS	Leland	Ext.- Leland/MS	Leland	Ext.- Leland/MS	Leland	Ext.- Leland/MS	Leland
Base	171.57	202.77	57.92	33.76	74.82	43.60	127.94	74.57	0.63	0.63	10.84	6.32	0.0247	0.0247
r = 0.02	148.61	165.24	30.88	20.78	77.69	52.27	87.55	58.92	0.53	0.53	4.71	3.17	0.0338	0.0338
r = 0.04	158.75	182.15	43.42	27.36	75.71	47.68	106.43	67.05	0.58	0.58	7.30	4.60	0.0286	0.0286
r = 0.08	186.71	226.57	73.97	39.62	74.78	40.04	151.77	81.29	0.67	0.67	15.47	8.29	0.0219	0.0219
δ = 0.02	406.51	495.73	165.73	40.77	159.98	39.36	335.77	82.60	0.68	0.68	25.28	6.22	0.0153	0.0153
δ = 0.04	227.75	273.23	84.39	37.06	94.73	41.59	178.47	78.37	0.65	0.65	14.19	6.23	0.0195	0.0195
δ = 0.08	145.64	169.93	45.14	30.98	66.01	45.34	103.92	71.34	0.61	0.61	9.44	6.48	0.0308	0.0308
σ= 0.05	84.93	115.51	56.74	66.83	23.57	23.57	112.42	112.42	0.83	0.83	7.13	7.13	0.0034	0.0034
σ = 0.15	124.17	153.68	54.77	44.12	46.40	37.36	107.27	86.40	0.70	0.70	7.77	6.26	0.0124	0.0124
σ = 0.35	229.71	264.24	64.16	27.93	108.65	47.30	155.61	67.73	0.59	0.59	15.65	6.81	0.0406	0.0406
b = 0.05	161.48	202.77	76.72	47.50	44.13	27.34	158.65	98.24	0.78	0.78	14.36	8.89	0.0305	0.0305
b = 0.25	166.65	202.77	67.05	40.24	59.10	35.46	143.66	86.21	0.71	0.71	12.55	7.53	0.0274	0.0274
b = 0.75	175.34	202.77	50.97	29.06	87.73	50.05	115.05	65.60	0.57	0.57	9.54	5.44	0.0229	0.0229
τ = 0.15	195.76	202.77	39.61	20.25	124.03	63.34	78.72	40.24	0.39	0.39	5.67	2.90	0.0120	0.0120
τ = 0.25	185.38	202.77	52.22	28.16	95.15	51.34	107.63	58.04	0.53	0.53	8.47	4.57	0.0187	0.0187
τ = 0.45	154.75	202.77	58.73	37.94	59.18	38.25	143.61	92.79	0.71	0.71	12.99	8.39	0.0305	0.0304

Notes: "Ext.-Leland/MS" refers to the model developed with both investment timing flexibility and debt financing gains. "McD&S" refers to McDonald and Siegel (1986) model of the perpetual investment option and "Leland" to the Leland (1994) model with optimal debt financing and no investment flexibility. Base case used for all models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ = 0.06, volatility σ = 0.25, investment cost I = 100. For the Ext. Leland and Leland model use bankruptcy costs b = 0.5, tax rate τ = 0.35.Equity, debt, optimal leverage, optimal coupons and the credit spread are calculated at the investment trigger. Sensitivity analysis is with respect to the risk-free rate r, opportunity cost δ, volatility of unlevered assets σ, bankruptcy costs b, and the tax rate τ.

Table III: Meeting the debt constraint: Adjustment in the investment trigger versus adjustment in the coupon level

Reduction in the investment trigger
Pros: ▪ earlier receipt of investment benefits and of net benefits of debt Cons: ▪ foregone value of timing flexibility ▪ increase in the probability of default (thus reducing the expected tax benefits and increasing expected bankruptcy costs)
Reduction in the coupon level
Pros: ▪ lower default trigger (increases the periods where tax benefits will be received) Cons: ▪ decrease in the level of tax benefits

Table IV: The effect of growth options and financing constraints with finite investment option maturity

	Firm value				
	T = 2	*T* = 5	*T* = 10	*T* = 20	*T* = 50
No constraints					
No Growth Option	24.83	29.06	32.17	34.34	35.22
$\gamma = 0.10$					
$\sigma_C = 0.2$	36.02	39.38	41.99	43.79	44.52
$\sigma_C = 0.4$	41.38	44.33	46.71	48.38	49.08
$\sigma_C = 0.6$	48.05	50.54	52.70	54.32	55.03
$\sigma_C = 0.2$					
$\gamma = 0.1$	36.02	39.38	41.99	43.79	44.52
$\gamma = 0.3$	61.08	62.83	64.38	65.51	65.97
$\gamma = 0.5$	95.07	95.55	96.09	96.53	96.71
Max Debt = 50					
No Growth Option	21.03	24.74	27.44	29.33	30.08
$\gamma = 0.10$					
$\sigma_C = 0.2$	30.62	33.57	35.84	37.39	38.03
$\sigma_C = 0.4$	35.22	37.79	39.86	41.30	41.91
$\sigma_C = 0.6$	40.90	43.07	44.95	46.34	46.96
$\sigma_C = 0.2$					
$\gamma = 0.1$	30.62	33.57	35.84	37.39	38.03
$\gamma = 0.3$	52.07	53.62	54.97	55.95	56.34
$\gamma = 0.5$	81.14	81.57	82.04	82.42	82.57

Notes: Base case used models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ = 0.06, volatility σ = 0.25, investment cost I = 100, bankruptcy cost b = 0.5 and tax rate τ = 0.35. Firm values are calculated using a Markov-chain implementation with N =50 states for the growth option (with average impact γ and volatility σ_C) and a numerical lattice scheme for the investment option with dt = 0.5 years. Max. Debt refers to constraints on the total amount of debt that can be issued.

Table V: The effect of growth options and financing constraints on firm value and its components (option on unlevered assets and expected net benefits of debt)

	Firm value			Option on Unlevered Assets $E[V\text{-}I]$		Net Benefits of Debt (NB)	
	Ext.-Leland/MS	McD&S	Leland	Ext.-Leland/MS	Leland	Ext.-Leland/MS	Leland
No constraints							
No Growth Option	35.42	25.48	18.18	24.67	0.00	10.75	18.18
$\gamma = 0.10$							
$\sigma_C = 0.2$	44.81	32.24	31.56	31.23	13.37	13.58	18.18
$\sigma_C = 0.4$	49.34	35.86	37.50	35.02	21.23	14.32	16.26
$\sigma_C = 0.6$	55.18	41.01	44.94	40.35	30.29	14.83	14.65
$\sigma_C = 0.2$							
$\gamma = 0.1$	44.81	32.24	31.56	31.23	13.37	13.58	18.18
$\gamma = 0.3$	66.25	47.74	59.60	46.41	35.30	19.84	24.30
$\gamma = 0.5$	96.90	70.46	94.85	69.17	64.88	27.73	29.96
Max Debt = 75							
No Growth Option	32.70	25.48	18.18	23.92	0.00	8.78	18.18
$\gamma = 0.10$							
$\sigma_C = 0.2$	41.36	32.24	30.41	30.32	13.37	11.04	17.04
$\sigma_C = 0.4$	45.24	35.86	35.07	34.31	21.23	10.94	13.84
$\sigma_C = 0.6$	50.06	41.01	41.10	39.81	30.29	10.25	10.81
$\sigma_C = 0.2$							
$\gamma = 0.1$	41.36	32.24	30.41	30.32	13.37	11.04	17.04
$\gamma = 0.3$	61.03	47.74	56.16	45.31	35.30	15.71	20.86
$\gamma = 0.5$	88.66	70.46	87.40	68.33	64.88	20.23	22.52
Max Debt = 50							
No Growth Option	30.25	25.48	14.87	24.68	0.00	5.57	14.87
$\gamma = 0.10$							
$\sigma_C = 0.2$	38.27	32.24	26.58	31.25	13.37	7.03	13.21
$\sigma_C = 0.4$	42.13	35.86	31.76	35.03	21.23	7.10	10.53
$\sigma_C = 0.6$	47.08	41.01	38.23	40.36	30.29	6.72	7.94
$\sigma_C = 0.2$							
$\gamma = 0.1$	38.35	32.24	26.58	31.25	13.37	7.03	13.21
$\gamma = 0.3$	56.58	47.74	50.71	46.42	35.30	10.16	15.40
$\gamma = 0.5$	82.74	70.46	80.93	69.18	64.88	13.56	16.05

Notes: "Ext.-Leland/MS" refers to the model with both investment timing flexibility and debt financing gains. "McD&S" refers to McDonald and Siegel (1986) model of the perpetual investment option and "Leland" to the Leland (1994) model with optimal debt financing and no investment flexibility. Base case used for all models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ = 0.06, volatility σ = 0.25, investment cost I = 100. For the Ext. Leland and Leland model use bankruptcy costs b = 0.5, tax rate τ = 0.35. Growth option parameters have expected impact γ and volatility σ_C and are implemented using a Markov-chain with N =50 states. Max. Debt refers to constraints on the total amount of debt that can be issued.

Table VI: The effect of growth options and financing constraints on optimal capital structure, expected costs, expected leverage ratio and on expected credit spreads.

	Optimal capital structure									
	Expected Equity		Expected Debt		Expected Cost		Expected Leverage		Expected Credit Spread	
	Ext. - Leland/MS	Leland	Ext.- Leland/MS	Leland	Ext.- Leland/MS	Leland	Ext.- Leland/MS	Leland	Ext.- Leland/MS	Leland
No constraints										
No Growth Option	25.79	43.60	44.10	74.57	34.47	100.00	0.63	0.63	0.0247	0.0247
$\gamma = 0.10$										
$\sigma_C = 0.2$	32.57	43.61	55.71	74.58	43.47	86.63	0.63	0.63	0.0247	0.0247
$\sigma_C = 0.4$	34.34	39.01	58.74	66.71	43.75	68.23	0.63	0.63	0.0247	0.0247
$\sigma_C = 0.6$	35.57	35.14	60.84	60.09	41.23	50.29	0.63	0.63	0.0247	0.0247
$\sigma_C = 0.2$										
$\gamma = 0.1$	32.57	43.61	55.71	74.58	43.47	86.63	0.63	0.63	0.0247	0.0247
$\gamma = 0.3$	47.59	58.28	81.38	99.68	62.72	98.35	0.63	0.63	0.0247	0.0247
$\gamma = 0.5$	66.52	71.87	113.77	122.91	83.38	99.93	0.63	0.63	0.0247	0.0247
Max Debt = 75										
No Growth Option	42.37	43.60	29.00	74.57	38.67	100.00	0.41	0.63	0.0110	0.0247
$\gamma = 0.10$										
$\sigma_C = 0.2$	53.51	53.11	36.43	63.93	48.58	86.63	0.41	0.55	0.0109	0.0194
$\sigma_C = 0.4$	57.18	53.07	35.80	50.23	47.73	68.23	0.39	0.49	0.0104	0.0173
$\sigma_C = 0.6$	61.13	53.72	33.21	37.67	44.28	50.29	0.35	0.41	0.0097	0.0147
$\sigma_C = 0.2$										
$\gamma = 0.1$	53.51	53.11	36.43	63.93	48.58	86.63	0.41	0.55	0.0109	0.0194
$\gamma = 0.3$	78.25	81.02	51.67	73.50	68.89	98.35	0.40	0.48	0.0107	0.0154
$\gamma = 0.5$	110.69	112.41	66.08	74.93	88.11	99.93	0.37	0.40	0.0099	0.0115
Max Debt = 50										
No Growth Option	47.45	64.87	17.20	50.00	34.40	100.00	0.27	0.44	0.0061	0.0122
$\gamma = 0.10$										
$\sigma_C = 0.2$	59.97	69.90	21.69	43.32	43.39	86.63	0.27	0.38	0.0060	0.0105
$\sigma_C = 0.4$	63.97	65.88	21.84	34.11	43.68	68.23	0.25	0.34	0.0058	0.0096
$\sigma_C = 0.6$	67.67	63.37	20.59	25.15	41.18	50.29	0.23	0.28	0.0055	0.0080
$\sigma_C = 0.2$										
$\gamma = 0.1$	59.97	69.90	21.69	43.32	43.29	86.63	0.27	0.38	0.0060	0.0105
$\gamma = 0.3$	87.89	99.88	31.31	49.18	62.62	98.35	0.26	0.33	0.0060	0.0086
$\gamma = 0.5$	124.39	130.90	41.65	49.97	83.30	99.93	0.25	0.28	0.0057	0.0067

Notes: "Ext.-Leland/MS" refers to the model with both investment timing flexibility and debt financing gains. "McD&S" refers to McDonald and Siegel (1986) model of the perpetual investment option and "Leland" to the Leland (1994) model with optimal debt financing and no investment flexibility. Base case used for all models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ = 0.06, volatility σ = 0.25, investment cost I = 100. For the Ext. Leland and Leland model use bankruptcy costs b = 0.5, tax rate τ = 0.35. Growth options parameters have expected impact γ and volatility σ_C and are implemented using a Markov-chain with N =50 states. All values reported are time zero expected values. Max. Debt refers to constraints on the total amount of debt that can be issued.

Table VII (a): Differential information between debt and equity holders with respect to volatility

	Firm value	Inv. Trigger (V_I)	Bankruptcy Trigger (V_B)	Optimal Capital Structure at Investment Trigger V_I				
				Equity	Debt	Leverage	Coupon	Credit Spread
Base (σ(*e*)=σ(d)=0.25)	35.42	171.57	57.92	74.82	127.94	0.63	10.84	0.0247
σ(*d*) =0.1	71.81	119.89	72.72	18.39	184.34	0.91	13.61	0.0138
σ(*d*) =0.15	52.88	140.02	71.54	33.21	169.53	0.84	13.39	0.0190
σ(*d*) =0.2	42.03	157.33	66.68	51.85	150.92	0.74	12.48	0.0227
σ(*d*) =0.3	31.34	182.54	46.16	101.39	101.37	0.50	8.64	0.0252
σ(*d*) =0.35	28.87	190.32	33.66	128.48	74.29	0.37	6.30	0.0248
σ(*d*) =0.4	27.41	195.38	22.81	152.02	50.75	0.25	4.27	0.0241

Table VII (b): Differential information between debt and equity holders with respect to growth

	Firm value	Inv. Trigger (V_I)	Bankruptcy Trigger (V_B)	Optimal Capital Structure at Investment Trigger V_I				
				Equity	Debt	Leverage	Coupon	Credit Spread
Base (δ(e)=δ(d)=0.06)	35.42	171.57	57.92	74.82	127.94	0.63	10.84	0.0247
δ(d) =0	53.93	138.64	71.75	31.99	170.76	0.84	13.43	0.0186
$\delta(d)$ =0.02	46.58	149.32	69.51	42.32	160.43	0.79	13.01	0.0211
$\delta(d)$ =0.04	40.31	160.67	65.08	56.47	146.27	0.72	12.18	0.0233
$\delta(d)$ =0.08	31.92	180.87	48.35	96.60	106.17	0.52	9.05	0.0252
$\delta(d)$ =0.10	29.57	188.00	37.83	119.47	83.30	0.41	7.08	0.0250
$\delta(d)$ =0.12	28.07	193.02	28.16	140.37	62.39	0.31	5.27	0.0245

Base case used for all models: value of unlevered assets V =100, risk-free rate r = 0.06, opportunity cost δ = 0.06, volatility σ = 0.25, investment cost I = 100, bankruptcy costs b = 0.5, and tax rate τ = 0.35.Equity, debt, optimal leverage, optimal coupons and the credit spread are calculated at the investment trigger. Sensitivity analysis is with respect to debt holders perceived estimate of volatility σ (d) (panel a) or the opportunity cost δ (d) (panel b). A higher estimate of δ (d) implies lower growth rate of the unlevered assets.