Multiple attribute evaluation of company financial level applying soft methodology (fuzzy approach)

Zdeněk Zmeškal, Dana Dluhošová[1]

VŠB - Technical University of Ostrava, Faculty of Economics, Department of Finance, Sokolská 33, 701 21, Czech Republic, Zdenek.Zmeskal@vsb.cz, Dana.Dluhosova@vsb.cz

Abstract

The ranking of the companies according to financial characteristics is crucial problem of the financial decision-making. Paper describes an approach to the multiple attribute financial level evaluation. Conditions of financial decision-making are supposed to be only vaguely determined. Fuzzy methodology is applied and fuzzy simple additive weighting method type model is designed. It means financial indices, weights are fuzzy terms and the aspect of non-fixed preciousness of data is included. Fuzzy sets of T-number types, extension principle, ε-cut methodology are used. Methodology of ranking companies is suggested. The simplified illustrative example is introduced.

Keywords

Financial level, financial analysis, fuzzy methodology, decomposition procedure, fuzzy multiple attribute weighting method, companies ranking.

1 Introduction

Determination of the financial level or rating is one of crucial financial decision-making problem. Typical feature of financial decision-making is that conditions and data is possible give only vaguely. There are many aspects and reasons of imprecise in finance.

Precision – the requirement for high level of precision may cause accounting models to lose part of their relevance to the real world by ignoring some relevant items because of not precise measurement or their inclusion may increase the complexity of model.

Neglect – many decision models are concentrated on benefits that are not difficult to measure, or improved accuracy could ignore benefits that are difficult to measure such as reduce decisional effort including mental ones. Neglecting of vagueness may cause the analysis to be incomplete, unrepresentative and irrelevant.

Effectiveness – high levels of precision required by accounting and financial models are not unwarranted, but also uncertainty for an effective analysis.

Applicability – the call for precise measure that are difficult to obtain can hinder the applicability of financial models.

Precise data ability providing - the demand for precise input data may represent a primary reason for the uneasiness of the potential models users. The difficulty concerns of decision maker to provide precise data necessary for models to give reliable results. When precise data are not attainable, the decision makers are forced to resort or enrich the data.

Fixed level of accuracy – this assumption is often unrealistic, and may cast doubt about the usefulness of the aggregate totals. There is question whether variables estimated with different degrees of accuracy can be meaningfully aggregated.

[1] The paper was supported by the Grant Agency of the Czech Republic (GACR): 402/08/1234.

Predictive and descriptive power – incorporating the vagueness and ambiguity displayed in the accounting environment into financial models may improve the descriptive and predictive power of models.

Flexibility – managers usually make decisions without having a full range of accurate measurement. The ability to deal with those situations affords them great flexibility in comparison with traditional techniques.

The introduced features are basic arguments for application of indeterminacy models. Since this aspect is typical for financial decision-making, it might be fruitful to apply soft computing methodology. Fuzzy approach is one of possibilities characterized by a good way of simplification, interpretability and ability of implementation.

A survey of vaguely formulated problems in finance and accounting decision-making is in Siegel et al. (1995), and in financial engineering Riberio et al. (1999). We can generally distinguish and see two basic approaches in dealing with soft (fuzzy-stochastic) financial modelling. The first one concerns using a fuzzy measure in contrast with a probability measure where a sub-additive property is not fulfilled, see e.g. Cherubini (2001), Yoshida (2003). The second approach is based on assumption that input data (parameters, distribution functions) is possible to introduce only vaguely. Financial applications examples, except option valuation models, are e.g. Lai and Hwang (1993), Inuiguchi and Ramík (2000), Tanaka et al. (2000), Cherubini and Lunga (2001), Carlsson et al. (2002), Zmeskal (2005), Koissi and Shapiro (2006). Special attention in hybrid (fuzzy-stochastic) financial modelling is paid to option valuations, because of providing abundant and a lot of applications. Researchers has dealt with relatively new topic of fuzzy-stochastic option valuation models which are presented e.g. in Zmeskal (2001), Simonelli (2001), Yoshida (2002), Carlsson and Fuller (2003), Yoshida (2003), Muzzioli and Torricelli (2004), Wu (2005), Cheng et al. (2005), Yoshida et al. (2005), Liyan Han and Wenli Chen (2006), Zhang et al. (2006), Wu (2007), Muzzioli and Reynaerts (2007) and Thiagarajaha et al. (2007).

The purpose of the paper is to apply soft methodology in financial evaluation and ranking of companies. The input data preciousness will be especially regarded. Fuzzy multiple attribute decision making model will be proposed and applied.

2 Crisp multiple attribute decision-making models formulation

We can distinguish under multiple criteria decision making (MCDM) methodology, (i) discrete type approach which is represented by multiple attribute decision making (MADM) models and (ii) continuous type approach characterised as multiple objective decision making (MODM). The determining of financial company level is included under the first approach.

The problem of decision-making could be divided into two phases. The first phase concerns the aggregations of satisfaction for all criteria per decision alternative. The second phase determines either the ranking of alternatives or assigning alternatives to some groups.

Assume the problem for the first phase is described by, (i) alternatives A_i, the set of corporations, (ii) criteria C_j, each alternative is characterized by the set of characteristics, in finance by financial ratios (indices), (iii) utility function for every company U_i is an aggregation of criteria by multiple attribute utility function (MAUF),

$$U_i = U_i\left(C_1, C_2 .... C_n\right). \tag{1}$$

Many utility models have been studied. The most applied is the composition of partial utility functions u_{ij} assuming the independence of criteria and addition. The very well known type of this function is simple additive weighting method (SAWM) where w_{ij} are weights of criteria and numerical ratings (indices) of alternative A_i for criteria C_j are $r_{ij} = u_i(C_j)$, aggregated utility function $U_i = U_i(r_{i1}, r_{i2} .... r_{in})$ is transformed as follows,

$$U_i = \sum_j w_j \cdot r_{ij} / \sum_j w_j \tag{2}$$

Special model with credibility of data is following,

$$U_i = \sum_j w_j \cdot r_{ij} \cdot v_{ij} \Big/ \sum_j w_j \,, \tag{3}$$

where parameter v_{ij} means degree of preciousness (credibility) of data. If $v_{ij} = 1$, for every j, the problem is of traditional SAWM version.

3 Fuzzy simple additive weighting model (FSAWM) description

In this section we present model methodology of fuzzy simple additive weighting method (FSAWM) type under T-numbers and non-fixed input data preciousness. The model is of SAWM version, however data are introduced vaguely by fuzzy sets of T-number type and assumption of non-fixed level of input data accuracy is considered.

There are possible to distinguish under fuzzy multiple attribute decision making (FMADM) methodology three basic model variants;

(i) Fuzzy variables – weights utility functions are stated as fuzzy sets, for instance, ”leverage is approximately 40 % ”, ”return on equity is about 10 %”, etc.
(ii) Linguistic variable – linguistic terms are assigned fuzzy sets, for example, ”leverage is sufficient”, ”liquidity is low”, etc.
(iii)Real (crisp) variable – special case of fuzzy models, fuzzy sets are of singleton type, it means real numbers and the model is deterministic.

Definition 1. *A fuzzy set* (depicted with tilde) is commonly defined by a membership function (μ) as representation from E^n (Euclid n-dimensional space, n > 1) to a set of E^1 specially to the interval of [0;1], $\tilde{s} \equiv \mu_{\tilde{s}}(x)$, where $\tilde{s}$ is fuzzy set, x is vector and $x \in X \subset E^n$, $\mu_{\tilde{s}}(x)$ is membership function.

Very fruitful and powerful instrument, which might be used, for calculating a function of fuzzy sets is extension principle, see (Zadeh (1965)).

Definition 2. The *extension principle* is derived by sup min composition between fuzzy sets $\tilde{r}_1 ... \tilde{r}_n$ and $\tilde{s} = f(\tilde{r}_1 ... \tilde{r}_n)$ as follows. Let $f : E^n \to E^1$, then membership function of fuzzy set $\tilde{s} = f(\tilde{r}_1 ... \tilde{r}_n)$ is defined by

$$\mu_{\tilde{s}}(y) \equiv \tilde{s} = \sup_{\substack{x1......xn \\ y = f(x1......xn)}} \min[\ \mu_{\tilde{r}1}(x_1)......\mu_{\tilde{r}n}(x_n)],\ x_i, y \in E^1 \tag{4}$$

The fuzzy multiple attribute decision-making (FMADM) models may be commonly expressed by extension principle as follows,

$$\tilde{U}_i \equiv \mu_{\tilde{U}i}(u_i) = \sup_{v;ui = f(yj,xij)} \min_j \left[\mu_{\tilde{w}j}(y_j); \mu_{\tilde{r}ij}(x_{ij}) \right], \tag{5}$$

where v = (y_1 ..... y_n; x_{i1} .....x_{in}) is vector and $\mu_{\tilde{U}i}(u_i), \mu_{\tilde{w}j}(y_j), \mu_{\tilde{r}ij}(x_{ij})$ are membership functions of $\tilde{U}_i, \tilde{r}_{ij}, \tilde{w}_j$.

Formulation of fuzzy utility function is following,

$$\mu_{\tilde{U}i}(u_i) = \sup_{v;u_i=\sum_j y_j \cdot x_{ij} \cdot z_{ij} / \sum_j y_j} \min_j [\mu_{\tilde{w}j}(y_j); \mu_{\tilde{r}ij}(x_{ij}); \mu_{\tilde{v}ij}(z_{ij})] \tag{6}$$

The solution method of the model described is not possible to get generally analytically and thus approximate procedure is to be applied. Procedure selection depends mainly on fuzzy set types and fuzzy operation between fuzzy sets.

Definition 3. *T-number* is fuzzy set meeting preconditions of normality, convexity, continuity and closeness and being defined as quadruple $\tilde{s} = (s^L, s^U, s^\alpha, s^\beta)$, where $\phi(\mathrm{x})$ is non-decreasing function and $\psi(\mathrm{x})$ is non-increasing function as follows, set of T-numbers is denoted by $F_T(E)$,

$$\tilde{s} \equiv \mu_{\tilde{s}}(x) = \left\{ \begin{array}{ll} 0 \quad for \quad x \le \mathrm{s^L} - s^\alpha; \phi(\mathrm{x}) \quad for \quad \mathrm{s^L} - s^\alpha < \mathrm{x} < \mathrm{s}^L; \\ 1 \quad for \quad \mathrm{s^L} \le x \le s^U; \ \psi(\mathrm{x}) \quad for \quad \mathrm{s^U} < \mathrm{x} < \mathrm{s^U} + s^\beta; \\ 0 \quad for \quad x \ge \mathrm{s^U} + s^\beta \end{array} \right\}. \tag{7}$$

Definition 4. The *linear T-number* is defined so as T-number where functions $\phi(\mathrm{x})$ and $\psi(\mathrm{x})$ are linear, $\phi(\mathrm{x}) = \frac{x - (s^L - s^\alpha)}{s^\alpha}$, $\psi(\mathrm{x}) = \frac{(\mathrm{s^U} + s^\beta) - x}{s^\beta}$, and is depicted as quadruple $\tilde{s} = (s^L, s^U, s^\alpha, s^\beta)$, set of linear T-numbers is denoted by F_{TL} (E).

Under T-numbers three approaches of solution FSAWM problem formulated by (6) might be used, (i) analytical solution by application of extension principle, (ii) approximate fuzzy algebraic operations procedure, (iii) approximate ε-cut procedure. Because analytical solution is not mostly usable this approach is not explained.

3.1 Approximate fuzzy algebraic operation procedure

Under this procedure the $\tilde{U}_i$ value is computed by approximate fuzzy algebraic operations between T-numbers. Because of application aspects and paper purpose, operations are described for linear T-number. The FSAWM model (6) under suppositions said could be formulated,

$$\tilde{U}_i = \tilde{w}_1 \otimes \tilde{r}_{i1} \otimes \tilde{v}_{i1} \oplus \tilde{w}_2 \otimes \tilde{r}_{i2} \otimes \tilde{v}_{i2} .....\tilde{w}_n \otimes \tilde{r}_{in} \otimes \tilde{v}_{in} \tag{8}$$

Definition 5. The operations of fuzzy addition and fuzzy multiplication are defined in accordance with Bonnisone (1982) for positive linear T-numbers as follows,

(i) *Fuzzy addition* $\left| \oplus, \tilde{\Sigma} \right|$ for $\tilde{\mathrm{s}}, \tilde{\mathrm{r}} \in \mathrm{F_{TL}}(\mathrm{E})$, for $\tilde{\mathrm{s}}; \tilde{\mathrm{r}} > 0$,

$$\tilde{s} \oplus \tilde{r} = \left(s^L; s^U; s^\alpha; s^\beta\right) \oplus \left(r^L; r^U; r^\alpha; r^\beta\right) = \left(s^L + r^L; s^U + r^U; s^\alpha + r^\alpha; s^\beta + r^\beta\right),$$

(ii) *Fuzzy multiplication approximation* $\left| \otimes; \tilde{\Pi} \right|$ for $\tilde{\mathrm{s}}, \tilde{\mathrm{r}} \in \mathrm{F_{TL}}(\mathrm{E})$, for $\tilde{\mathrm{s}}; \tilde{\mathrm{r}} > 0$,

$\tilde{s} \otimes \tilde{r} = \left(s^L \cdot r^L; s^U \cdot r^U; s^L \cdot r^\alpha + r^L \cdot s^\alpha - s^\alpha \cdot r^\alpha; s^U \cdot r^\beta + r^U \cdot s^\beta - s^\beta \cdot r^\beta\right)$,
where fuzzy set is *positive fuzzy set*, if for every $x \in \sup \tilde{s}$, $x>0$, and $\sup \tilde{s} = \{x \in X; \mu_{\tilde{s}}(x) > 0\}$.

Remark Operation of fuzzy addition is in accordance with extension principle. Fuzzy multiplication is approximate operation, because the result of multiplication is not linear T-number under extension principle rule.

Advantage of the method is in using a simple and few calculations; the results are in the same class of fuzzy sets. Disadvantage consists in confining approach applicability on special types of fuzzy sets, (linear) fuzzy number type, and getting only approximate results.

3.2 Decomposition principle

There are often in solving practical and complex problems under fuzzy environment such troubles that analytic solution according to extension principle is not available. Assuming a fuzzy set is of fuzzy number type (the T-number type as well) there is possible to solve function of fuzzy numbers $\tilde{s} = f(\tilde{r}_1 \ldots \tilde{r}_n)$ in accordance with extension principle as decomposition procedure.

Definition 6. The ε*-cut* of the fuzzy set $\tilde{s}$, depicted $\tilde{s}^\varepsilon$, is defined as follows.

$$\tilde{s}^\varepsilon = \{x \in E^n; \mu_{\tilde{s}}(x) \geq \varepsilon\}, \tag{9}$$

where ${}^-s^\varepsilon = \inf\{\mu_{\tilde{s}}(x); x \in E^n, \mu_{\tilde{s}}(x) \geq \varepsilon\}$, ${}^+s^\varepsilon = \sup\{\mu_{\tilde{s}}(x); x \in E^n, \mu_{\tilde{s}}(x) \geq \varepsilon\}$.

Definition 7. Decomposition principle (Resolution identity) is defined as follows,
$\mu_{\tilde{s}}(y) = \sup_{\varepsilon}\{\varepsilon \cdot I_{\tilde{s}^\varepsilon}; y \in \tilde{s}^\varepsilon\}$, for any $y \in E^n$ and $\varepsilon \in [0;1]$,
where $\tilde{s}^\varepsilon = [{}^-s^\varepsilon, {}^+s^\varepsilon]$ is ε - cut, ${}^-s^\varepsilon(x) = \min_{x \in \tilde{x}^\varepsilon \subset E^n} f(x)$, ${}^+s^\varepsilon(x) = \max_{x \in \tilde{x}^\varepsilon \subset E^n} f(x)$. Here $I_{\tilde{s}^\varepsilon}$ is characterization function,

$$I_{\tilde{s}^\varepsilon} = \begin{cases} 1 \text{ if } y \in [{}^-s^\varepsilon, {}^+s^\varepsilon] \\ 0 \text{ if } y \notin [{}^-s^\varepsilon, {}^+s^\varepsilon] \end{cases}. \tag{10}$$

It is apparent that applying the Definition 7 the function of fuzzy numbers could be transformed and solved as several mathematical programming problems for ε by this way,

$$\max (\min) \ s \equiv {}^+s^\varepsilon, ({}^-s^\varepsilon),$$

$$\text{s.t. } s = f(x_1 \ldots . x_n),$$

where $x_i \in \left[{}^-x_i^\varepsilon, {}^+x_i^\varepsilon\right]$ for $i \in \{1;2 \ldots\ldots n\}$, and $\varepsilon \in [0;1]$.

Advantage of procedure is in generalised application possibility, mainly for convex, normal fuzzy numbers. Disadvantage consists in computation difficulty.

3.3 Ranking and similarity of fuzzy sets

Because the utility value $\tilde{U}_i$ is the fuzzy set, to solve the second problem phase means a ranking of fuzzy sets or assigning them into groups. Solutions concern a choice of binary relation types. There are many approaches, (i) binary preference relations, (ii) fuzzy scoring relations, (iii) fuzzy mean and spread, (iv) linguistic methods, see for instance Dubois (1980), Tsukumoto (1983), Tong (1984), Ramik (1986), Lee (1988), Chen (1989), Ramik (1996), Ramik (2001).

The purpose of determining corporate financial level is to assign the corporations in groups. The similarity measure is useful conception, which could be used; one of examples is discrete inclusion measure $I_{ik}(\tilde{U};\tilde{Z}_k)$:

$$I_{ik}(\tilde{U};\tilde{Z}_k) = \left|\tilde{U} \cap \tilde{Z}_k\right| / \left|\tilde{U}\right|, \tag{11}$$

where $\tilde{Z}_k$ is kth category of financial corporate level, $\tilde{U} \cap \tilde{Z}_k = \bigcup_x \min\left[\mu_{\tilde{U}_i}(x), \mu_{\tilde{Z}_K}(x)\right]$.

Further $\left|\tilde{U}\right|$ is card of fuzzy set, which is defined for discrete and continuous case as follows, $\left|\tilde{U}\right| = \sum_x \mu_{\tilde{U}}(x)$ or $\left|\tilde{U}\right| = \int_x \mu_{\tilde{U}}(x)\, dx$.

It is apparent, the measure $I_{ik} \in [0;1]$. If $I_{ik} = 1$, then corporation is fully a member of particular category. If $I_{ik} = 0$, then company is not member of category.

4 Illustrative example

Now the simplified model of FWSAM type with non-fixed data preciousness (6) and under linear T-number (Definition 4), parameterised by linguistic variables, solved by decomposition procedure (Definition 7) will be described and solved.

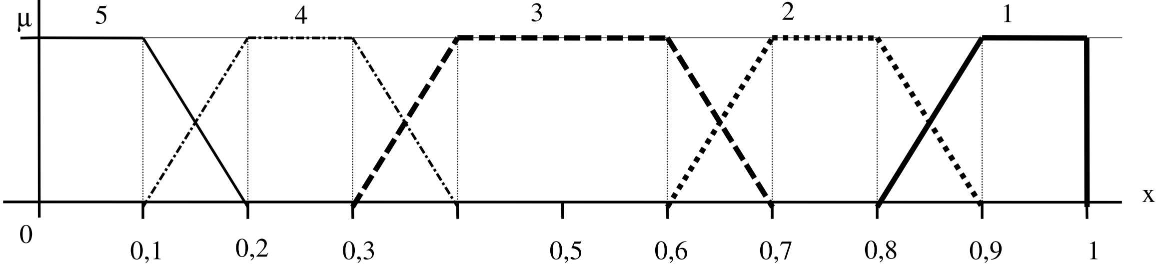
We assume we have three companies (A1, A2, A3) which might be assigned in five categories of financial level, which are articulated as follows, (i) excellent, (ii) good, (iii) middle, (iv) bad, (v) poor.

Further, we suppose the financial level is characterised through three criteria, (i) profitability, (ii) liquidity, (iii) leverage, weights of criteria and data preciousness are expressed linguistically and every term is equivalent with linear T-number. The particular shape is shown in Table 1 and Figure 1.

Table 1 Input linguistic parameters and linear T-number shape assigned

	Financial level	**Profitability, Liquidity, Data preciousness**	**Leverage**	**Weights**	**linear T-number**			
					a^L	a^U	a^α	a^β
1	excellent	high	low	significant	0,9	1	0,1	0
2	good	rather high	rather low	-	0,7	0,8	0,1	0,1
3	middle	middle	middle	middle significant	0,4	0,6	0,1	0,1
4	bad	rather low	rather high	-	0,2	0,3	0,1	0,1
5	poor	low	high	non-significant	0	0,1	0	0,1

Figure 1 Graphical representation of linear T-numbers



The financial level of corporations (A1, A2, and A3) including linguistic assignment of financial characteristic with weights and data preciousness shows Table 2 (For the sake of simplicity parameter of data preciousness is for every financial characteristic for particular company the same).

Table 2 Linguistic characteristics of companies financial level

Characteristics	**Profitability**	**Leverage**	**Liquidity**	**Data preciousness**
Weights	middle significant	significant	non-significant	-
Company A1	rather high	low	middle	rather high
Company A2	rather low	rather low	rather low	middle
Company A3	low	rather low	middle	rather low

Calculated results of fuzzy utility functions in accordance with (6) and maximum inclusion measure I_{ik} according to (10) are shown in the following Table 3.

Table 3 Evaluation results of companies' financial level

Company	**Inclusion – similarity measure**						**Evaluation**
	$\mathbf{I_{i1}}$	$\mathbf{I_{i2}}$	$\mathbf{I_{i3}}$	$\mathbf{I_{i4}}$	$\mathbf{I_{i5}}$	**max** $\mathbf{I_{ik}}$	
A1	0,2641	0,8338	0,5818	0,0117	0	0,8338	good
A2	0	0	0,4153	0,975	0,3	0,975	bad
A3	0	0	0	0,4884	0,6141	0,6141	poor

From results it is possible to judge that on the FSAWM method basis including data preciousness determining, the financial level of corporation A1 is good, A2 bad, A3 poor.

It is interesting to compare these results with the same problem but without considering the data different preciousness. The calculations show (detailed data are not for the sake of conciseness introduced) that financial level of company A1 is excellent (max I= 0,7326), A2 is middle (max I=0,80567) and A3 is also middle (max I=0,93255).

This simple example illustrates the significant difference as far as the results are concerned.

5 Conclusion

The methodology of FMADM shows the possibility to describe realistically conditions and circumstances of decision making. There are many approaches used. From implementation point of view the FSAWM method is simple and useful. As was explained there are many aspects of vagueness. The weights, utility functions could be stated vaguely. In the paper the aspect of non-fixed preciousness input data were stressed and modelled vaguely and FSAWM model was modified. This aspect is often neglected, but in financial decision it is present and very significant and therefore it could be suitable to model and include degree of preciousness data parameter in fuzzy multiple attribute decision models.

References

BONISSONE, P.P.: *A Fuzzy Sets Based Linguistic Approach, Theory and Application*, In: Gupta, M.M., Sanchez, E. (1982): Approximate Reasoning in Decision Analysis. North-Holland,

CARLSSON, Ch., FULLER, R., MAJLENDER, P. (2002) *A possibilistic approach to selecting portfolios with highest utility score. Fuzzy Sets and Systems*, 13, 2002, pp 13-21

CHEN, S.J., HWANG,C.L. (1989): *Fuzzy Scoring of fuzzy number* - A direct comparison index, Kansas State University

Cheng Few-Lee, Tzeng Gwo-Hsiung, Wang Shin-Yun, 2005. A Fuzzy Set Approach for Generalized CRR Model: An Empirical Analysis of S&P 500 Index Options. *Review of Quantitative Finance and Accounting*, 25, pp. 255–275.

CHERUBINI, U., LUNGA, G. D. (2001) *Fuzzy Value-at- risk*: Accounting for market Liquidity. Economic Notes by Banca dei Paschi di Siena SpA, vol. 30, no 2, 2001, pp 293-312

COPELAND, T.E, WESTON, J.F. (1988), *Financial theory and corporate finance*, Addison - Wesley.

DUBOIS, D., PRADE, H.,.(1980), *Fuzzy sets and systems*, Academic Press, New York, 9-35.

INUIGUCHI, M. RAMIK, J. (2000) *Possibilistic linear programming: a brief review of fuzzy mathematical programming and comparison with stochastic programming in portfolio selection problem, Fuzzy Sets and Systems*, 111, pp 3-28

LIYAN HAN, WENLI CHEN (2006). The Generalization of λ-Fuzzy Measures with Application to the Fuzzy Option. In: Wang, L. et al. (Eds.) Proceedings: Fuzzy Systems and Knowledge Discovery, LNAI 4223, pp. 762–765, Springer-Verlag Berlin Heidelberg.

MUZZIOLI, S., REYNAERTS, H. (2007) The solution of fuzzy linear systems by non-linear programming: a financial application. *European Journal of Operational Research* 177 1218–1231

MUZZIOLI, S., TORRICELLI, C. (2004). A multi-period binomial model for pricing options in a vague world. *Journal of Economic Dynamics and Control* 28, pp. 861-887.

RAMÍK, J., ŘIMANEK, J. (1986) *Inequality relations between fuzzy numbers and its use in fuzzy optimization. Fuzzy Sets and Systems*, 16

RAMÍK, J., VLACH, M., (2001), *Generalized concavity in fuzzy goal and decision analysis*. Kluwer Academic Publishing, Boston – Dodrecht – London

RIBEIRO, R. A. (1996): *Fuzzy multiple attribute decision-making. A review and new preference elicitation techniques. Fuzzy Sets and Systems*, No. , 151-18178

RIBERIO, R. A., ZIMMERMANN, H. J., YAGER, R. R., KACPRZYK, J. (1999) *Soft Computing in Financial Engineering*, Springer Verlag

SAKAWA, M., KATO, K., NISHIZAKI, I. (2003) *An interactive fuzzy satisficing method for multiobjective stochastic linear programming problems through an expectation model. European Journal of Operational Research*, Vol. 145/3, pp 665-672

SIMONELLI, M. R. (2001). Fuzziness in valuing financial instruments by certainty equivalents. *European Journal of Operational Research*, 135/2, pp. 296-302.

TANAKA, H, GUO, P. TURKSEN, I. B. (2000) *Portfolio selection based on fuzzy probabilities and possibility distributions, Fuzzy Sets and Systems*, 111, 387-397

THIAGARAJAHA, K., APPADOOB, S.S., THAVANESWARANC, A. (2007). Option valuation model with adaptive fuzzy numbers, *Computers and Mathematics with Applications* 53, pp. 831–841.

TONG, R.M., BONISSONE, P.P. (1984): *Linguistic solutions to fuzzy decision problems*. In: TMS/Studies in the Management Science, 20, Zimmermann, H.J., Elsevier Science Publishers B.V., North-Holland, p.323-334

YOSHIDA, Y. (2002). A discrete-time model of American put option in an uncertain environment. *European Journal of Operational Research* 151, 153-166.
YOSHIDA, Y. (2003). The valuation of European options in uncertain environment. *European Journal of Operational Research* 145, pp. 221-229.
YOSHIDA, Y., YASUDA, M., MALAGAMI JUN-ICHI, KURANO, M. (2005). A Discrete-Time American Put Option Model with Fuzziness of Stock Prices. *Fuzzy Optimization and Decision Making*, 4, pp. 191–207, Springer Science Business Media.
ZADEH, L. A. (1965), "*Fuzzy sets*", Information Control, 8 (3), 338-353.
ZHANG JINLIANG, DU HUIBIN, TANG WANSHENG (2006) Pricing R&D option with combining randomness and fuzziness. Computational intelligence, PT 2, Proceedings Book Series: Lecture notes in artificial intelligence, Volume: 4114, 798-808
ZMESKAL, Z. (2008). The application of the American real flexible switch options methodology – a generalised approach. *Finance a úvěr - Czech Journal of Economics and Finance,* Vol. 58, Issue**:** 5-6, pp. 261-275
ZMEŠKAL, Z. (2001) *Application of the fuzzy - stochastic Methodology to Appraising the Firm Value as a European Call Option, European Journal of Operational Research*, Vol. 135/2, pp 303-310
ZMEŠKAL, Z., (2002) *Company Financial Multiple Attribute Evaluation under Vagueness Conditions*. MOPGP, pp. 421-432, In. Multiple Objective and Goal Programming - Recent Developments, eds. T.Trzaskalik, J. Michnik, Springer-Verlag , 2002.
ZMESKAL, Z. (2005). Value at risk methodology under soft conditions approach (fuzzy-stochastic approach). *European Journal of Operational Research*, 161 (2), pp. 337-347.