A Wavelet-heterogeneous Index of Market Shocks for assessing the Magnitude of Financial Crises*

(Work in progress)

Christophe Boucher† Gregory Jannin‡ Bertrand Maillet§ Hélène Raymond-Feingold¶

- April 2009 -

Abstract

An accurate quantitative definition of financial crisis requires a universal and robust scale for measuring market shocks. Following Zumbach et al. (2000) and Maillet et Michel (2003), we propose a new quantitative measure of financial disturbances, which captures the heterogeneity of investor horizons – from day traders to pension funds. The indicator resides on a multi-resolution analysis of market volatility, each scale corresponding to various investment horizons and different data frequencies. This new risk measure, called “Wavelet-heterogeneous Index of Market Shocks” (WhIMS), is based on the combination of two methods: the Wavelet Packets Sub-band Decomposition and the constrained Independent Component Analysis (see Kopriva and Seršić, 2007 and Lu and Rajapakse, 2005). We apply this measure on the French stock market (high frequency CAC40) to date and gauge the severity of financial crises. A state separation of financial disturbances based on the WhIMS and conditional probabilities of the HMC-MLP model is then performed using a Robust SOM based on a hybrid model combining a Hidden Markov Chain and MultiLayer Perceptrons.

*We thank Thierry Chauveau, Benjamin Hamidi, Patrick Kountchou, Paul Merlin and Jean-Philippe Médecin for encouragements and a kind support when preparing this work. We here acknowledge Thierry Michel, Madalina Olteanu, Patrick Rouset and Joseph Rynkiewicz for active participations in previous collaborations on the subject. We should finally mention the excellent preliminary research assistance by Rachid Bokreta, Caroline Barrault and Alexander Subbotin on related topics. The third author thanks the Europlace Institute of Finance for financial support. Preliminary version (work in progress): do not quote or circulate without an explicit permission. The usual disclaimers apply.

†A.A.Advisors-QCG (ABN AMRO), Variances and University of Paris-1 (CES/CNRS). E-mail: christophe.boucher@univ-paris1.fr.
‡A.A.Advisors-QCG (ABN AMRO), Variances and University of Paris-1.
§A.A.Advisors-QCG (ABN AMRO), Variances and University of Paris-1 (CES/CNRS and EIF). Correspondance to: Dr. Bertrand B. Maillet, CES/CNRS, MSE, 106 br de l’hôpital F-75647 Paris cedex 13. Tel: +33 144078283/47. E-mail: bmaillet@univ-paris1.fr.
¶University of Paris-10 (Economix/CNRS). E-mail: helene.raymond-feingold@u-paris10.fr.
A Wavelet-heterogeneous Index of Market Shocks
for assessing the Magnitude of Financial crises

1 Introduction

Extreme price movements in financial markets are of primary concern not only for practitioners but also for policy makers and monetary authorities by their potential consequences on macroeconomic and financial stability. A clear understanding of financial stability requires an accurate measure of financial turbulence. It is important to know when such pressure occurs and what its intensity is to detect, identify and compare the severity of different crises. Scales to measure the severity of extreme risks are frequently used in geology, meteorology, astronomy and other sciences: the Guttenberg-Richter, Saffir-Simpson, TORRO, and Torino scales. By contrast, there is still no consensus about a scale measure of market shocks. Few attempts have been proposed in the literature (Cf. Mishkin and White, 2003) aiming to provide comparisons of financial crises across markets, or over time in a single market. But financial crises are still most often measured by simple binary variables based on extreme values (1, 2 or 3 standard deviations above the mean value) of one or few underlying financial variables.

The objective of this paper is to propose a new quantitative measure of financial crises, which we called the WhIMS (for Wavelet-heterogeneous Index of Market Shocks) and to identify regimes in financial turbulence, i.e. normal and crisis states. A non-linear classification using a robust Kohonen map analysis based on the WhIMS and conditional probabilities of a hybrid Hidden Markov Chain – Multilayer Perceptron model is performed to characterise and identify these regimes.

The WhIMS quantifies the intensity of market turbulence. This measure of market disturbances is a refinement of the so-called Scale of Market Shocks (SMS) by Zumbach et al. (2000) and the Index of Market Shocks (IMS) by Maillet and Michel (2003) In short, its construction is based on an analogy with the so-called Richter scale used for measuring earthquake intensity. The main step consists in applying a Robust Wavelet Packets Sub-band Decomposition constrained Independent Component Analysis (RWPSD-cICA) first to decompose the return volatility at different time scales (see Lu and Rajapakse, 2005; Kopriva and Seršić, 2007) and, secondly, to extract independent factors resulting of the decomposition. Then, we fit volatility extreme values from a Generalized Pareto Distribution (GPD) based on the L-moment method.

The motivation behind the wavelet decomposition is the existence of traders with different time horizons. The heterogeneous market hypothesis was first introduced by Dacorogna et al. (1993) which observed a scaling law relating time horizon and size of price movements (Müller et al., 1990). Short-term traders, such as day-traders, are constantly watching the market; they re-evaluate the situation and execute transactions at a high frequency horizon. Long-term investors, such as pension funds, may look at the market less frequently. Intuitively, a quick price increase of 0.5% followed by a quick decrease of the same size, for
example, is a major event for the intra-day trader but a non event for central banks and pension funds. Sometimes, price movements may have a certain influence on the timing of both day-traders and long-term investors’ transactions and investment decisions. The WhIMS gauges the severity of these markets fluctuations that impact both day-traders, hedge funds, financial and banking institutions, mutual funds, central banks, and pension funds.

The wavelet decomposition consists in a decomposition of a signal into its set of basis functions (wavelets), analogous to the use of sines and cosines in Fourier analysis. These basis functions are obtain from dilations or contractions (scaling), and translations of the mother wavelet. The main advantage of wavelet analysis is the ability to decompose the data into several time scales and ability to handle nonstationary data, localization in time, and the resolution of the signal in terms of the time scale of analysis. Wavelet analysis, popular in disciplines such as signal processing and medical sciences, is an increasing used tool in economics and finance. Wavelets have been used in a variety of financial applications, such as outlier testing (Greenblatt, 1995), the modelling of non-stationary processes (Ramsey and Zhang, 1997) and long-memory processes (Jensen, 1999), time-series decomposition (Ramsey and Lampart, 1998), forecasting (Stevenson, 2001), scaling analysis (Gencay et al., 2002), CAPM and ICAPM testing (Gencay et al., 2003 and 2005; Fernandez, 2006) and analysis of higher-order systematic co-moments (Galagedera and Maharaj, 2008). A thorough discussion of the use of wavelets in economics and finance can be found in the survey articles by Ramsey (1999 and 2002).

Conventional time series analysis focuses exclusively on a time series at a given scale but some recent researchs explore the behavior of financial time series, especially volatilities, at different time scales. The HARCH models of Muller et al. (1997) and Dacorogna et al. (2001) belongs to the ARCH family but differs from ARCH-type processes by considering the volatilities of returns measured over different interval sizes. The HARCH model has the ability to capture the asymmetry in the interaction between volatilities measured at different frequencies such that a coarsely defined volatility, which captures the view and actions of long-term traders, predicts a fine volatility (short-term traders) better than the way around. Ait-Sahalia et al. (2005) proposed an estimator of the volatility based on overlapping subsampling schemes and an appropriate combination of two realized volatilities computed at two different time scales. More recently, Ait-Sahalia et al. (2006) have generalized the Two Scales estimator to a multiple time scales estimator that combines realized volatilities computed at more than two return frequencies and reaches the same asymptotic efficiency of the kernel-based estimator. Our empirical study is also related to the increasing literature that emphasizes the structural breaks in financial volatility and more generally in financial time series (see Lamoureux and Lastrapes, 1990; Andreou and Ghysels, 2002).

The outline of this paper is as follows. Section 2 introduces the Independent Component Analysis and the Wavelet Packet Sub-band Decomposition. Section 3 presents the Generalized Pareto Distribution estimated based on L-moments estimation method. Section
4 described the WhIMS computation. Section 5 presents empirical estimations. Section 6 proposes to identify regimes in financial turbulence based on a hybrid model combining a Hidden Markov Chain and a Multilayer Perceptron. We discuss the main results along with future directions in conclusion.

2 A Time-scale Decomposition of the Volatility

We decompose the volatility into different time scales by applying a new wavelet and factor analysis method: the Wavelet Packets Sub-band Decomposition Independent Component Analysis (WPSD-cICA). The Independent Component Analysis (ICA) is well-known in Signal Processing and applied in various fields such as biomedicine, speech and telecommunication signals. The ICA is a factor analysis method that extracts independent components from a set of multidimensional observations. This method corresponds to an extension of the Principal Component Analysis which provides orthogonal directions. However, the ICA knows several limits. Indeed, Chang and Zhang (2005) demonstrate that the independence property of the extracting independent components is not always verified. Some extensions have been proposed to overcome this principal limit of the traditional ICA. Throughout this paper, we will introduce a new method which combines two decomposition and factor analysis methods: the Wavelet Packet Sub-band Independent Composition Analysis (WPSD-ICA - Kopriva and Sersi, 2007) and the constrained Independent Component Analysis (cICA - see Lu and Rajapakse, 2005). The result of this mix gives the WPSD-cICA.

2.1 An Introduction to the Wavelet Analysis

Our approach to multi-scale analysis is based on the wavelet transform of the original time series of stock returns. The definition of WhIMS resides on the estimation of the vector of volatilities at different time scales \( \sigma^{(1)}, \ldots, \sigma^{(m)} \). A natural tool for this purpose is wavelet analysis. It is widely used in signal processing in order to decompose a given time series \( x(t) \) called “signal” into a hierarchical set of approximations and details (multi-resolution analysis), and to decompose energy of the signal on scale-by-scale basis.

Many methods of wavelet analysis exist. The most well known is the Discrete Wavelet Transform (DWT) which is generally used in image analysis. However, we use the WPSD or "tree-structures decomposition" method which is an extension of the Discrete Wavelet Transform by improving the frequency selectively. The bases are the same. All wavelet share the same basic construction plan: the choice of wavelet mother and the choice of the level of decomposition (Percival and Walden, 2000, pp.135 - 145). The algorithm algorithm is a compromise between decomposition and smoothing techniques. At each level of decomposition \( j \) approximations \( A_j \) and details \( D_j \) are built. The initial signal can be represented in the following way:
where \( N \) is the last level of decomposition and \( x(t) \) the original time series. The suitable level of decomposition can be chosen either endogenously, i.e. based on the properties of the signal \( x(t) \) or using \textit{a priori} hypothesis.

Any decomposition is based on the so-called wavelet mother function \( \Psi \) and the associated wavelet father function \( \Phi \). \( \Psi \) is used to define the details while \( \Phi \) is used to define approximations. Then, the wavelet transform \( C \) consists in calculating a "resemblance index" between the signal and the wavelet function located at position \( b \) and of scale \( a \):

\[
C(a, b) = \int_{-\infty}^{+\infty} x(t) \frac{1}{\sqrt{a}} \Psi \left( \frac{t - b}{a} \right) dt.
\]

This transformation can be applied at continuous or discrete scales. The DWT and the WPSD preserve the main features of the Continuous Wavelet Transform (CWT). For a properly chosen wavelet function, the synthesis of the original signal can be done:

\[
x(t) = \sum_{j=1}^{N} \sum_{k=1}^{K} C(j, k) \Psi_{j,k}(t),
\]

with \( j \) corresponding to decomposition levels and \( k \) to positions of wavelet on the time axis.

As for the choice of the wavelet mother function, we use the most well known Daubechies scaling filter. There are no universal criteria for the choice of the types of the wavelet. However, some drawbacks can occur in the multi-resolution analysis if the wavelet filter doesn’t match with the characteristics of the time series. Concerning the choice of the level of decomposition, no such easy rule is defined. We select in our application a number of level allowing us to determine very long-term fluctuations.

In this paper, we present another wavelet analysis method: the Wavelet Packet Sub-band Decomposition (WPSD). It is a generalization of the DWT that offers a large set of decomposition structure. Wavelet Packets were introduce for a better treatment of the non-stationnarity of data. We mobilize this tool to decompose and to reconstruct return trajectories of the CAC40 returns.

### 2.2 Wavelet Packets Sub-band Decomposition constrained Independent Component Analysis with L-negentropy

We propose a combination of two Blind Source Separation methods (WPSD and cICA) using a robust version of the negentropy criterion called L-negentropy in order to form the Wavelet Packet Sub-band Decomposition constrained Independent Component Analysis.
(WPSD-cICA). It allows us to overcome two of the main limits of the traditional ICA: the independence property that is not verified and no dimension data reduction.

A standard ICA algorithms is a statistical technique to extract non-Gaussian and statistically independent source signals given only the observed or measured data. The task is to find \( \mathbf{W} \) such that the distribution of \( \mathbf{Y} \) is as far from Gaussian as possible and statistically independent.

\[
\mathbf{Y} = \mathbf{WX}
\]

where \( \mathbf{X} \) represents the vector of measured signals, \( \mathbf{W} \) represents the demixing matrix and \( \mathbf{Y} \) represents the estimated vector of the unknown vector of the source signals \( \mathbf{S} \).

Kopriva and Seršić (2007) show that the source source signals can be dependent, although some of their sub-components are independent. They generalize this basic ICA model by a Sub-band Decomposition ICA (SD ICA). Thus, source signals can be represented as:

\[
\mathbf{S} = \mathbf{s}_1 + \mathbf{s}_2 + \cdots + \mathbf{s}_L
\]

where \( \mathbf{s}_k \), with \( k = [1, \cdots , L] \), are narrow-band sub-components. The standard ICA algorithms can be applied to a selected (reduced) set of \( k \) sub-components in order to learn demixing matrix \( \mathbf{W} \) so that \( \mathbf{y}_k = \mathbf{Wx}_k \). The sub-components are extracted by the Wavelet Packet algorithm (WP - see Kopriva and Seršić, 2007; Kotnik and Kacic, 2007). The result of the decomposition is often represented as a tree of \( J \) levels and of \( N \) nodes as follows:

\[
x^j_{kn}(t) = \sum_l f^j_{knl}\phi_j(t),
\]

where \( j \) represents scale level, \( k \) represents sub-band index, \( n \) represents the sensor index, \( l \) represents shift index, \( \phi_j(t) \) is the chosen wavelet function (or atom) at the time \( t \) and \( f^j_{knl} \) are decomposition coefficients.

A first method for selecting sub-bands (or least dependent components) is to use the Kullback-Leibler Information Criterion (KLIC - see Gouriéroux and Jasiak, 2008), which is a special case of \( \alpha \)-divergence measures (see Csiszar, 2005), and defined by \( I(\mathbf{x}) \) as such:

\[
I(x^j_k) = - \int f_{x^j_k}(u) \ln[f_{x^j_k}(u) f_0(u)^{-1}] \, du,
\]

where \( f_{x^j_k}(.) \) is the "true" density function of the observations and \( f_0(.) \), the approximant density, is the standardized Gaussian distribution.

Since the estimation of \( I(.) \) requires the knowledge of the true probability distribution function, this measure is difficult to estimate. It is usually approximated in practice with an Edgeworth expansion of the Gaussian approximant density, based on the computation of (conventional) order cumulants, such as:

\[
I(x^j_k) \approx \frac{1}{12} \kappa_3(x^j_k)^2 + \frac{1}{48} \kappa_4(x^j_k)^2
\]
where \( \kappa_i(x_k^j) \) is the \( i \)-th order conventional cumulant of \( x_k^j \).

However, the validity of such an approximation may be rather limited due to far-from-normality features of the true density but also to instability of Conventional algebraic moments.

We propose hereafter a new approximation of the contrast function based on TL-moments (Elamir and Seheult, 2003), by assuming that the distributions \( y_j \) are either Generalized Pareto Distributions (GPD) or Gaussian distributions. This allows us to take into consideration a large range of observed financial distributions. Moreover, in order to increase the capability of reducing the data dimension and improve the quality of source separation, we integrate prior information and an additional form of constraints into the ICA contrast function \( I(\cdot) \), as in Lu and Rajapakse (2005).

We assume in fact that \( y_j \) has a GPD, with characteristic parameters \( \xi_j \) and \( \alpha_j \). Then the L-negentropy WPSD-cICA algorithm is an iterative fixed-point algorithm with the following update for unmixing matrix \( W \), for each \( w_j \) being the \( j \)-th row vector of \( W \) such as:

\[
w_j^* = \text{ArgMax}_{w_j \in \mathbb{R}^N} \{12^{-1}[\frac{2\alpha_j^2(1+\xi_j)}{(1-\xi_j)^2(1-2\xi_j)(1-3\xi_j)}]^2 \]
\[
+48^{-1}[\frac{3\alpha_j^2(3+\xi_j+2\xi_j^2)}{(1-\xi_j)^2(1-2\xi_j)(1-3\xi_j)(1-4\xi_j)} - .18]^2 \}
\]

s.t.: \( g(y_j^k) \leq 0 \),

with \( \xi_j \) and \( \alpha_j \) being the estimated TL-moments characteristic tail-index and location parameters of the normalized GPD distribution of \( y \), and where the inequality constraint writes:

\[
g(y_j^k) = [g_1(y_{k1}^j), g_2(y_{k2}^j), \cdots, g_n(y_{kn}^j)]',
\]

with \( g_i(y_{ki}^j) = \epsilon(y_{ki}^j, s_{ki}^j) - \epsilon_i \), and where \( \epsilon(y_{ki}^j, s_{ki}^j) \) being the error term of the extraction, and \( \epsilon_i \) a threshold.

A remarkable property of the proposed L-negentropy algorithm is not only its robustness to outliers but also the high speed of convergence in the iterations.

### 3 Extreme Value Theory: Estimation of a Generalized Pareto Distribution with a Generalized Method of TL-moments

Recent attempts for modelling distributions in a multivariate framework are built on the order-statistics, for calibrating a Bernstein Copula in Baker (2008) or for defining extreme comovements using L-moments in Serfling and Xiao (2007). The later, which are linear functions of the expectations of order statistics, were introduced under this name by Sillitto (1951) and comprehensively reviewed by Hosking (1989). As so-called U-statistics,
one of their main advantage over the conventional \( (C-) \) moments is that their empirical counterparts are less sensitive to the effects of sampling variability, since linear functions of the ordered data. They are shown to provide more robust estimators of higher moments that the traditional sample moments and have then found wide applications in fields where extreme events matter, such as meteorology, hydrology and also earthquake analysis with the Richter Scale (See Thompson et al., 2007). More precisely, L-moments are defined as certain linear functions of the Probability Weighted Moments and can characterize a wider range of distributions compared to the usual moments. Indeed, they exist whenever the mean of the distribution does, even though some conventional moments do not. Moreover, they are easy to compute and reliable estimators for extreme distributions.

To compute the WhIMS, we use a GPD estimated with Generalized Method of TL-moments (Bali, 2003a and 2003b and Hosking, 2007). The GPD of the WhIMS block \textit{maxima}, denoted here \( \sigma \), for the sake of simplicity is given by:

\[
G_\xi(\sigma) = \begin{cases} 
1 - \left[ 1 + \xi \frac{(\sigma - \nu)}{\alpha} \right]^{-\xi^{-1}} & \text{if } \xi \neq 0 \\
1 - \exp \left[ -\frac{(\sigma - \nu)}{\alpha} \right] & \text{otherwise}, 
\end{cases}
\]

for every \( \sigma \in \mathcal{D} \), defined by:

\[
\mathcal{D} = \begin{cases} 
\sigma = -\infty; \nu - \frac{\alpha}{\xi} & \text{if } \xi < 0 \\
\sigma = \nu & \text{if } \xi = 0 \\
\sigma = \nu + \frac{\alpha}{\xi}; +\infty & \text{if } \xi > 0.
\end{cases}
\]

This allows us to deduce the cumulants of order 3 and 4 for a GPD normalized distribution:

\[
\kappa_3 = \frac{2\alpha^3(1+\xi)}{(1-\xi)^2(1-2\xi)(1-3\xi)} \\
\kappa_4 = \frac{3\alpha^4(3\xi+2\xi^2)}{(1-\xi)^2(1-2\xi)(1-3\xi)(1-4\xi)} - 3.
\]

Moreover, the first three TL-moments, as a function of the three characteristic parameters of a GPD distribution, are given, for every \( (s, t) \in \mathbb{N}^2 \), by:

\[
\lambda_1^{(s,t)} = \frac{v - \xi}{\alpha} + \frac{(1+s+t)!}{t!} \frac{\Gamma(t-\xi+1)}{\Gamma(2+s+t-\xi)} \xi \\
\lambda_2^{(s,t)} = \frac{2(t+1)!}{3(t+2)!} \frac{\Gamma(3+s+t-\xi)}{\Gamma(t-\xi+1)} \alpha, \\
\lambda_3^{(s,t)} = \frac{3(t+2)!}{3(t+2)!} \frac{\Gamma(4+s+t-\xi)}{\Gamma(t-\xi+1)} (1+\xi) \alpha,
\]

where \( \lambda_r^{(s,t)} \) is the \( r \)-th TL-moment of truncation order \( (s, t) \), \( v \in \mathbb{R} \) the location parameter, \( \alpha \in \mathbb{R}_+ \) the scale parameter, \( \xi \in \mathbb{R} \) the tail index and \( \Gamma(a) = \int_0^{+\infty} t^{a-1} e^{-t} dt \) is the Gamma function.
4 The Wavelet-heterogeneous Index of Market Shocks

The measure of market disturbances we propose is a refinement of the so-called Scale of Market Shocks (SMS) by Zumbach et al. (2000) and the Index of Market Shocks (IMS) by Mailet and Michel (2003, 2005). Zumbach et al. (2000) proposes an indicator called the Scale of Market Shocks (SMS) computed directly from the dynamics of asset prices. The SMS is an indicator of market volatility, analogous to the Richter scale in geophysics (Richter, 1958). The Richter scale is a measure of the logarithm of the total energy liberated during an earthquake. According to recent evidence, the probability of occurrence of large earthquakes grouped within temporal clusters of high seismic activity obeys the inverse power law (see Bak et al., 2002 and Mega et al., 2003). The distribution of the time intervals between one earthquake and the next is an inverse power law. By definition, a one point increase on the corresponding log₂ scale means an event, which is twice as unlikely (or twice as intense). It has been widely documented that the probability of large price changes decays as a power law, which results in the fat-tailed return distributions (Gabaix et al., 2003). The analogy with the Richter scale for financial markets is a complex matter mainly because the notion of “total energy” is hard to specify: many indicators can be potential candidates for this. Zumbach et al. (2000) characterize the size of shocks on the financial market by a continuous operator, mapping price volatility to a logarithmic scale. The latter stands as a counterpart of mechanical work, i.e. the rate of change of energy in time. The corresponding formula is:

\[ SMS_t = \alpha \log_{10} [P(v_t)]^{-1} \] (12)

with \( P(v_t) \) the distribution function associated with the indicator of aggregated realized volatility \( v_t \) and \( \alpha \), a scaling parameter. This indicator is designed to be used as a basis for definition of crisis, which is a period when it is higher than some pre-specified threshold. Note that the SMS by Zumbach et al. (2000) accounts not only for the size but also for the scale of fluctuations. The basic idea is that the price dynamics is driven by the actions of heterogeneous market participants, operating at different frequencies (Muller et al., 1997). To make this idea clear, compare an institutional investor who operates over medium and long term targets and a small private investor exploiting short term market fluctuations. Hardly do they share the same opinion on what is trend and fluctuation on the market. Namely, what is seen as conjectural fluctuation by the first may often be regarded as a trend by the second. However, sometimes they would agree to characterize the situation on the market as a crisis or a crash. The underlying intuition is that volatility is characterized not only relatively to particular time series, but also relatively to particular scales of observations. Economically, these scales can be seen as the decision and portfolio adjustment horizons of investors. The existence of multiple scales in volatility, whatever the underlying model could be, motivates designing an indicator, which characterizes the overall vector of volatilities at different scales. The Scale of Market Shocks (SMS), proposed in Zumbach et al. (2000) for the currency markets, takes the form of an average
realized volatility across different scales:

$$SMS_t = \sum_{k=1}^{m} w_k f(\sigma_t^{(k)})$$

(13)

where $\sigma_t^{(k)}$ is the volatility of returns at scale $k$, $w_k$ is the convolution kernel measuring the contribution of scale $\tau_k$ to the overall volatility and $f(\cdot)$ is some properly chosen mapping function. Let $M_k$ be the number of observations of price available for any period of time of length $\tau_k$. An estimate of (realized) price volatility for a given scale is then given by:

$$\sigma_t^{(k)} = \frac{1}{\sqrt{M_k}} \sum_{i=1}^{M_k-1} r_{t,\delta}^2$$

(14)

with $r_{t,\delta}^2$ represents squared log-returns computed for time intervals $\delta = k(M_k - 1)^{-1}$. As the SMS was constructed for inhomogeneous tick-by-tick data, the formula (2) could not be applied directly and the authors used smoothed volatilities, computed over moving windows, of the form:

$$\sigma_t^{(k)} = \int_{t-2\tau(k)}^{t} K(\phi) w_{\phi}^{(k)} \sigma^{(k)}(\phi) d\phi$$

(15)

with $K(\cdot)$ an appropriately chosen kernel function and $w_{\phi}^{(k)}$ a scale dependant weight. A potential problem is that the scaling method suggested in Zumbach and Muller (2001) gives no idea about the range of scales to be considered and their relative importance. They assert that the choice of the convolution kernel, denoted in formula (15), is not a crucial issue and it suffices to take a function which satisfies some regularity conditions and tends to zero at the edges of the range of frequencies, which is 15 minutes - 64 days in their case. To our knowledge, this choice is made a priori and is not based on any economic notion or on statistical properties of the time series. The assumption that the mass point, or the most important frequency is located in the centre of the range, is also doubtful. Another important drawback is that (15) does not account for the possible interdependence between scales.

Maillet and Michel (2003, 2005) adapted the multi-scale approach to the stock market with regards to the complexities due to the discontinuity of the time of trades. They use a different approach for computation and aggregation of volatilities at multiple frequencies, applying the Principal Components Analysis (PCA). The new indicator, called Index of Market Shocks (IMS), was used for detection and comparison of severity of different crises. The IMS is designed for the stock market data and uses a different method of multi-resolution analysis.

Instead of computing volatilities for different scales by successively moving a window of fixed length, the IMS uses different frequencies of sampling within each interval to obtain
the scale components of volatility. The aggregation scheme is more delicate: it is based on new information contained in each scale relatively to other scales. Instead of scaled volatilities themselves, principal components are used in the definition of IMS:

\[ IMS_t = \sum_{i=1}^{q} w_i \log_2 [1 - F(c_i)] \]  

(16)

where \( F(.) \) is the cumulative density function, \( c_i, ..., c_q \) are normalized principal components (or factors underlying multi-scale volatility), and \( w_i \) is the weight of each component determined as the portion of variance of the data explained by the corresponding component. Using logarithm in base 2 gives a clear economic sense to the indicator: a one point increase of IMS corresponds to the energy vector twice as unlikely.

Several shortcomings of the IMS should be mentioned. First, the method of estimating volatilities at different scales by varying sampling frequency has one important drawback which becomes very important if applied to low frequency data. If the length of computation window is fixed and volatilities are calculated from the samples with different number of points, the results of estimation will be different in terms of statistical error and may become incomparable. Second, the definition of vector \( c_i, ..., c_q \) in (5) is too restrictive: it relies on linear decorrelation and may not have the power to identify factors if they are not symmetrically distributed (Loretan, 1997) or if the underlying model is not a simultaneous linear mix. Third, the weighting scheme is simply based on the eigenvalues of the PCA mixing matrix, so it does not differentiate between factors influencing short or long scales. So volatilities at different horizons are treated as if they were all of equal importance to investors.

The WhIMS formula is:

\[ WhIMS_t = -\alpha \sum_{i=1}^{K} \{ w_k \log_2 [1 - F(\sigma_i^2)] \} \]  

(17)

where \( F(.) \) is the cumulative density function, \( \sigma_i \) represents each independent factors of volatilities at different time scales, \( w_k \) is the weight of each Independent Components obtain from a Wavelet Packets Sub-band Decomposition constrained Independent Component Analysis, and \( \alpha \) a scaling constant.

The WhIMS is computed in two steps. The first one is a combination of two methods which are applied on volatility signals: a Wavelet Packets and a Subband Decomposition Independent Component Analysis (See Lu and Rajapakse, 2005 and Kopriva and Seršić, 2007). This method permits to determine quantitatively the range of frequencies computing the various volatilities instead of fixing it arbitrarily (Maillet and Michel, 2003 and 2005). The difference with the traditional ICA is that this method allows us to get components truly independent and to introduce a constraint in the signals to reduce the data dimension. Moreover, this tool is more appropriate than the Principal Component Analysis used in the original IMS computation when data are non linear and non Gaussian. The
second step consists in fitting cumulative density function of Independent Components with a Generalized Pareto Distribution based on L-moments estimation method (which are order-statistic cumulants). Indeed, this step guarantees the WhIMS not to depend anymore on the hypothesis of Log-normality of volatilities.

Finally, the algorithm of computing WhIMS can be summarized as follows. Firstly, on a moving window of fixed length (equal to a power of two) perform the Wavelet Packet Subband Decomposition of returns; secondly, we reconstruct the trajectories of returns for each scale; thirdly, we transform data for all scales to log-squared returns; fourthly, we compute constrained Independent Components and their weights; fifthly, we fit the generalized Pareto distribution for the tails of each Independent Component; and sixthly, we compute the WhIMS using formula (17).

The next section contains empirical results obtained for the French stock market data.

5 Empirical Results

In a first time, we study the statistical properties of our time series. In a second time, we apply the WPSdICA tool to the returns of the CAC40 in order to decompose the source signal into several ones of different time scales and to reduce the dimension of this components in Independent Components.

5.1 Data

We estimate the WhIMS using CAC40 high frequency data between 1997 and 2007. The CAC40 index, computed by Euronext France, is available at 30 seconds frequency. However, as widely documented in recent financial literature (see Corsi et al., 2001, and Oomen, 2005, for a detailed discussion), significant microstructure effects can appear at the highest frequencies. So we restrict our analysis to long enough 15 minutes intervals. This period takes account of two financial major events: the Asian crises of 1998 and the crises of the technological values from 2000 to 2003. Below, you can see an illustration of our data.

The illustration of the CAC40 and its returns show clearly that turbulence periods (1998 with the Asian crises, 2000 to 2003 with the Internet crises and September 2001) alternate with calm periods. This Figure shows that CAC40 returns are characterized by volatility clusters. Table 1 presents some statistical properties of the CAC40 returns. These statistics suggest that the CAC40 returns are characterized by an asymmetric distribution with fat tails. The maximum positive and negative variation of prices are respectively equal to 4.70% and -6.49%. Moreover the kurtosis is equal to 23. These features testify the use of statistical methods that take account of extreme events in the distribution of the CAC40 returns.
5.2 An Illustration of the Heterogeneity Concept

To visualize the heterogeneity phenomenon, we represent the stock price of the CAC40 at different frequencies (30 minutes, 1 day and 1 week). The Figure 2 shows that a shock in the short term will probably have no consequences on an investors that anticipate in low frequencies such as th pension funds. But in crisis periods, all the financial operators are concerning by falling stock prices. We illustrate this fact in the Figure 3. This Figure represents the state of volatility: we give the value 1 if the volatility is an extreme event, 0 otherwise. Indeed, we can see that during the 2000-2003 period, all the investors whatever their investment horizon are touched by the crisis. In other words, a crises is a multiscale phenomenon.

5.3 Application of the WPSDcICA to the CAC40 Returns

The basic idea of the wavelet analysis is to decompose a signal into a certain number components of different time scales. We apply this method to the 30 minute intraday returns of the CAC40. After decomposing the source signal, we build new trajectories of returns at different time scales. Each level of decomposition \( j \) corresponds to a frequency of \( 2^j \) days. The Figure 4 represents the different rebuild trajectories of the CAC40 returns in different time scales going to one hour to 6 months. The original time series of intraday returns can be reconstructed by summing up the details and the approximation according to formula (6).

5.4 The WhIMS applied to Intraday CAC40 Data

We identify regimes of financial crises that are defined when our risk measure exceeds the arbitrary threshold corresponding to a 90\% confidence level. We applied the WhIMS algorithm to our data in order to detect and identify market turbulence during the studying period using the formula (17).

The WhIMS highest values during the period 2000-2003 mainly show that the French market is characterized by a strong instability over these years (Figure 5), whereas it is rather quite calm since 2003, with a rebirth of volatility in the recent months corresponding to the latest credit events. Moreover, the Index accounts for the multi-scale features of market volatility and is proven to be robust to the distributional properties of data.

6 A Robust Hybrid HMC-MLP Modelling of Financial Crises

We finally introduce a model for the Wavelet-heterogeneous Index of Market Shocks (WhIMS) based on a Hidden Markov Chain (HMC) and Multilayer Perceptrons (MLP). Following Maillet et al. (2004), we propose to identify regimes in financial turbulence,
\textit{i.e.} normal and crisis states. Over long periods, financial time series exhibit important breaks in their behavior and properties. Abrupt changes may be due to several reasons, such as bankruptcies, burst of bubbles, structural changes in business cycles conditions (\textit{i.e.} disinflation), or wars and related events. One way to capture structural breaks is to use HMC. This method initially proposed by Baum and Petrie (1966), has been widely applied in various fields, including speech recognition, signal processing, DNA recognition and financial time series.

We first present the Autoregressive HMC Models and estimate the model based on a French stock market index (CAC40 Index) to compare the prediction performance of the HMC-MLP model to classical linear and non-linear models. A state separation of financial disturbances based on the WhIMS and conditional probabilities of the HMC-MLP model is finally performed using a Robust SOM.

6.1 Research of a HMC-MLP Model

Let us consider \((Y_t)_{t \in \mathbb{N}}\) the observed time series and let \((X_t)_{t \in \mathbb{Z}}\) be a homogeneous Markov chain defined by its state space \(S = \{s_1, \ldots, s_N\}, \ N \in \mathbb{N}^*\) and the \((N \times N)\) transition matrix \(\mathbf{A}\) with \(a_{ij} = P(X_{t+1} = s_i | X_t = s_j), (i,j) = [1, \ldots, N]^2\). If we suppose, with no loss of generality, that the chain state space is the canonical basis of \(\mathbb{R}^N\) then, we note \(v_{t+1} = X_{t+1} - E[X_{t+1}|X_t]\), then an autoregressive hidden Markov chain model has the following form:

\[
\begin{cases}
X_{t+1} = \mathbf{A}X_t + v_{t+1} \\
Y_{t+1} = F_{X_{t+1}}(Y_{t,t-p+1}) + \sigma_{X_{t+1}} \varepsilon_{t+1}
\end{cases}
\]

where \(Y_{t,t-p+1}\) defines the vector \((Y_{t-p+1}, \ldots, Y_t)\), \(F_{X_{t+1}} \in \{F_{s_1}, \ldots, F_{s_N}\}\) is an autoregressive function of order \(p\), \(\sigma_{X_{t+1}} \in \{\sigma_{s_1}, \ldots, \sigma_{s_N}\}\) is a real strictly positive number, \((\varepsilon_t)_{t \in \mathbb{N}}\) are independently, identically distributed according to a standardized Gaussian law. In particular, we apply nonlinear autoregressive functions, such as the multilayer perceptrons, to consider the so-called hybrid HMC-MLP models (Hidden Markov Chain - Multilayer Perceptron models).

We focus on daily values of the WhIMS, computed from the CAC40 French stock market index, from July 9th, 1987 until April 30th, 2008 (5,243 observations). We first investigated whether the series (estimated using the Quasi-Maximum Likelihood method) is “regime switching” and what type of structure should be chosen and, secondly, if we can distinguish different market behaviors by using a hybrid Hidden Markov Chain – Multilayer Perceptron model (HMC-MLP). The linear model and the one hidden-layer perceptron that were considered in a preliminary study showed that the significant lags were 1, 2, 4, 5 and 6. We fixed this input vector and we let both the number of experts and hidden units vary up to three. In the end, two configurations were selected and investigated, on the basis of two empirical criteria: the first architecture is the one to have the smallest mean squared error and the second is the one with the “best” transition matrix. In the latter case, the “best” transition matrix is the one empirically generating
the most stable segmentation of the series, that is its trace divided by its dimension is the closest to one.

First, we compare the estimated HMC-MLP model with the results of a linear ARMA (see Brockwell et al., 2002) and of a Multilayer Perceptron. After investigating all ARMA\((p, q)\) models, for \(p\) and \(q\) in a specified range (both lower than 10), an AR(6) model minimizing the BIC criterion was selected. The research of the Multilayer Perceptron is done using the Baum-Welch algorithm (see Baum et al., 1970) for Maximum Likelihood parameter estimation and the Viterbi algorithm (see Viterbi, 1967) to compute the most probable sequence. The “best” model is chosen to minimize the BIC criterion and the “Statistical Stepwise” algorithm is used to eliminate the non-significant connections, once the “dominating” perceptron is found. The final selected model has two hidden units, twelve parameters and the input vector time dimension goes up to 6.

Preliminary comparisons of the mean squared error (in Appendix) suggest there is no special interest in considering a HMC-MLP model for forecasting the exact value of the WhIMS as there is no sensible improvement when compared to the linear model or to the perceptron (results available upon request). Intuitively, this negative result is not surprising when recalling that sudden and violent movements in the market are clearly difficult to predict, and that crises are often due to exogenous and unforecastable events (e.g. for the September 2001 terrorist attack, bank failures). Nevertheless, a state representation of the turbulence is of interest since a qualitative assessment (crisis versus non-crisis period) is worthy and decisive for financial applications. We will focus hereafter on studying the state separation of the possible series considering a three-state Hidden Markov Chain model.

### 6.2 A Three-state HMC-MLP Model

The research of a three state model having the most significant segmentation of the series leads to select an architecture with the following estimated transition matrix:

\[
\hat{A} = \begin{pmatrix}
.91 & .06 & .03 \\
.02 & .83 & .15 \\
.02 & .13 & .85
\end{pmatrix}
\]

The three experts are three hidden units perceptrons. The associated estimated variances are, respectively, .77, .36 and .24 and the mean squared error is 1.03.

The interpretation of a three-state model is however not straightforward than a simple two-state one (crisis versus non-crisis), but preliminary results suggest that a 3-state model allow better discrimination of crisis and non-crisis periods than a 2-state model. We present hereafter a robust Kohonen classification for describing the behavior of the Wavelet-heterogeneous Index of Market Shocks subject to the three experts. The basic idea of Kohonen map is to display high dimensional information on a plane by putting together observations with related characteristics (see Kohonen, 2000, for the description of Self-Organizing Maps algorithm). But, since during the learning process the outliers
influence the model by moving the neurons toward the outliers, we use a robust version of the SOM (see Maillet et al., 2005).

We consider the WhIMS and the conditional probabilities related to the three perceptrons as input variables. Thus each cell in the map will incorporate information about the current state of the market but also some information about the dynamics of WhIMS trajectories through the conditional probabilities of belonging to each of the three states. The three probabilities come from the HMC-MLP model and thus correspond to a denoized estimate of the market conditions. The non-linear classification done by the Kohonen algorithm is then based on the observed WhIMS values, the modelled ones and the dynamics of the series.

We first conduct a hierarchical classification performed on the map (results available upon request). If we aggregate the information contained in this Kohonen map, by cutting the classification tree and only keep three clusters, we note that the WhIMS values migrate through clusters, from small to high values; the first and the last cluster are homogeneous and correspond, respectively, to periods of calm and crisis respectively (see Figure 6). The intermediate cluster - associated to medium WhIMS values and the third expert, but also to mixtures of experts mentioned above - is less homogeneous. This is even more obvious if we cut the classification tree at five clusters: no significant change can be noted regarding the first and the last clusters, but the middle ones are splitted according to different expert combinations (results available upon request).

The separation between high values of the WhIMS - associated with the first expert - and low values of the Index - linked to the predictions of the second expert - are quite insensitive to the number of clusters. Performing a Kohonen map analysis has here the major advantage that the cut between regimes is less arbitrary, and show that a clear separation can be made based on the WhIMS value and the value of conditional probabilities to be in some states. Note that state separation is more effectively achieved with the observed WhIMS instead of the predicted one.

Finally, we investigate the behavior of returns in the three identified clusters. Table 2 presents some performance measures of portfolios corresponding to each identified regime: the annualized mean return, the volatility, the Sharpe ratio (a risk-adjusted return measure) and the frequencies of up and large down returns. We compare the characteristics of the constructed portfolios between each identified cluster and between different HMC-MLP models of financial disturbances. Indeed, we consider four different measures of financial turbulence: the WhIMS, the Index of Market Shocks proposed by Maillet and Michel (2003), the VIX index and the volatility, which correspond respectively to an aggregate measure of the implied volatility of a wide range of S&P 500 options and to the one-year daily volatility of returns.

Differences between the three clusters clearly reflect the various disturbance regimes from high return-low volatility (cluster 1) to low return-high volatility (cluster 3), with an intermediate state which is undetermined and corresponds to a wider range of WhIMS. The mean conditional expected daily returns are not significantly different, but the cumu-
lated differences on the whole sample amount to large discrepancies between conditional performances. Moreover, the state separation based on the WhIMS leads to a better discrimination of market conditions than those based on the IMS, the VIX or the classical volatility.

The three portfolios derived from the WhIMS are shown on the Figure 7: the series "State 1" ("State 2", "State 3") is built considering the benchmark returns when the period is classified in the first (second, third) cluster of the three-category classification and a zero return when classified elsewhere, either in the second (first, first) clusters or third (third, second). While the first state (first cluster) is clearly associated with low WhIMS and low return, the second cluster is more in line with volatile markets, whilst the third cluster is generally associated with large drops in the market. These results suggest that jumps or regime switching in volatility could potentially strongly affect portfolio allocation.

7 Conclusion

We proposed a quantitative measure of financial crisis - the WhIMS, based on a multi-resolution analysis of market volatility. This indicator aims to characterize volatility as perceived by different types of financial agents who have various investment horizons. It can be computed both for large samples of low-frequency data and high frequency data. The algorithm of computing WhIMS is based on a wavelet analysis to decompose the volatility at different time scales combined with a factor analysis to identify volatility factors at each scale. The WhIMS does not rely on the log-normality of these factors, making it more robust to distributional properties of the data.

We established a quantitative definition of crises based on the distribution of the WhIMS to date events on financial markets, as well as to compare the relative severity of these events. Finally, we proposed to model the WhIMS in a HMC-MLP framework to account for potential regime switching in financial turbulences. A non-linear classification, such a robust Kohonen map analysis, based on the WhIMS and conditional probabilities of the HMC-MLP model allows to identify and characterise stock market conditions. A major issue remains to determine statistically and no more economically the number of regimes in the HMM (e.g. Gassiat, 2002; Olteanu, 2006). From an asset allocation and risk-management perspective, a promising direction of future research would be to investigate how the identification of market condition regimes based on expert conditional probabilities altogether with WhIMS values affects the investor’s portfolio optimisation problem.
8 References


Figure 1: Time Evolution of the CAC40 and the CAC40 returns

Source: Euronext. CAC40 30 minutes intraday data from January 2nd, 1997 to December 31st, 2007. The chart on the top represents the evolution of the CAC40 whereas the chart on the bottom is an illustration of the CAC40 intraday returns. Computations by the authors.
Figure 2: Time-Series Evolution of the CAC40 at Different Calendar Frequencies

Source: Euronext. CAC40 30 minutes intraday data during the period 06/11/2002 to 07/19/2002. The first chart represents the 30 minutes CAC40 prices, the second one is the 1 day CAC40 prices and the third and last chart illustrates the 1 week CAC40 prices. Computations by the authors.
Figure 3: Time-series Evolution of the CAC40 Return Volatility States
(0 if calm period, 1 if in the 90% extreme)

Source: Euronext. CAC40 30 minutes intraday data from the January 2nd, 1997 to the December 31st, 2007. This chart represents the state volatility in the y-axis, the frequency in the z-axis (30 minutes to one month) and time in the x-axis. Computations by the authors.
Figure 4: Evolution of Some Levels of Decomposition of CAC40 Returns

Scale 1 (about 1 hour)

Scale 2 (about 2 hours)

Scale 3 (about ½ day)

Scale 4 (about 1 day)

Scale 5 (about 2 days)

Scale 6 (about 4 days)

Scale 7 (about 8 days)

Scale 8 (about 16 days)

Scale 9 (about 32 days)

Scale 10 (about 1 quartely)

Scale 11 (about 1 semester)

Sources: Euronext. CAC40 30 minutes intraday data from the January 2nd, 1997 to the December 31st, 2007. The charts represent eleven levels of decomposition of the CAC40 intraday returns from a Wavelet Packets constrained Independent Component Analysis. Computations by the authors.
Figure 5: Evolution of the CAC40 Index and the Wavelet-heterogeneous Index of Market Shocks

Sources: Euronext. CAC40 30 minutes intraday data from the January 2nd, 1997 to the December 31st, 2007. The chart on the top represents the evolution of the CAC40 in brown and crises in gray. The chart on the bottom represents the WhIMS applied to CAC40 in black line and the 90% threshold in red line. Computations by the authors.
Figure 6: Clustering of the Kohonen Map of States of Market Turbulences (Three-category Tree)

State 1: "Normal" Market Conditions
State 2: High WhIMS
State 3: Highest WhIMS (Crises)

Source: Bloomberg. Weekly CAC40 Index data between January 1st, 1987 and April 30th, 2008. Clusters resulting from a Three-category Hierarchical classification of the four normalized inputs of the Kohonen Map: the WhIMS (first (x,y) coordinate), and the three-state conditional probabilities (the three following (x,y) coordinates: P₁, P₂ and P₃). The Bold line represents the mean Code Vector of each cell. Computations by the authors.
Figure 7: Cumulated Return Series Related to States of Turbulence of the CAC 40

Source: Bloomberg. Weekly CAC40 Index data between January 1st, 1987 and April 30th, 2008. "State 1" ("State 2", "State 3") corresponds to a series of cumulated returns (base 100 in January 1st, 1990) when the WhIMS and the conditional expert probabilities are classified in the first (second, third) cluster on a three-category string (semi-logarithmic scale). Computations by the authors.
Table 1: Statistics of Intraday Returns on the CAC40

<table>
<thead>
<tr>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Median</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Negative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6.49%</td>
<td>4.70%</td>
<td>.00%</td>
<td>.36%</td>
<td>.00%</td>
<td>-.21</td>
<td>22.99</td>
<td>48.47%</td>
</tr>
</tbody>
</table>

Source: Euronext. CAC40 30 minutes intraday data during January 2nd, 1997 to December 31st, 2007. Computations by the authors.
Table 2: Comparisons between Market Characterizations based on a HMM-MLP Modelling of the WhIMS, IMS, VIX and Volatility

<table>
<thead>
<tr>
<th>State</th>
<th>CAC40</th>
<th>Frequency</th>
<th>Return</th>
<th>Volatility</th>
<th>Sharpe ratio</th>
<th>Up</th>
<th>Large Down</th>
</tr>
</thead>
<tbody>
<tr>
<td>All States</td>
<td>CAC40</td>
<td>100.00</td>
<td>5.24</td>
<td>20.56</td>
<td>8.46</td>
<td>54.38</td>
<td>10.00</td>
</tr>
<tr>
<td>State 1</td>
<td>WHIMS</td>
<td>63.68</td>
<td>25.50</td>
<td>15.50</td>
<td>1.42</td>
<td>60.54</td>
<td>4.09</td>
</tr>
<tr>
<td>IMS</td>
<td></td>
<td>44.54</td>
<td>20.85</td>
<td>15.58</td>
<td>1.11</td>
<td>59.12</td>
<td>4.14</td>
</tr>
<tr>
<td>VIX</td>
<td></td>
<td>35.78</td>
<td>11.36</td>
<td>17.66</td>
<td>0.45</td>
<td>55.76</td>
<td>7.58</td>
</tr>
<tr>
<td>Volatility</td>
<td></td>
<td>31.68</td>
<td>15.25</td>
<td>20.70</td>
<td>0.57</td>
<td>57.34</td>
<td>9.56</td>
</tr>
<tr>
<td>State 2</td>
<td>WHIMS</td>
<td>29.84</td>
<td>-15.69</td>
<td>24.43</td>
<td>-0.79</td>
<td>44.93</td>
<td>17.75</td>
</tr>
<tr>
<td>IMS</td>
<td></td>
<td>35.03</td>
<td>-2.25</td>
<td>19.84</td>
<td>-0.29</td>
<td>51.85</td>
<td>12.04</td>
</tr>
<tr>
<td>VIX</td>
<td></td>
<td>37.08</td>
<td>7.99</td>
<td>15.51</td>
<td>0.29</td>
<td>57.14</td>
<td>6.71</td>
</tr>
<tr>
<td>Volatility</td>
<td></td>
<td>36.32</td>
<td>2.28</td>
<td>19.36</td>
<td>-0.06</td>
<td>51.94</td>
<td>7.46</td>
</tr>
<tr>
<td>State 3</td>
<td>WHIMS</td>
<td>6.49</td>
<td>-47.44</td>
<td>35.49</td>
<td>-1.44</td>
<td>38.33</td>
<td>35.00</td>
</tr>
<tr>
<td>IMS</td>
<td></td>
<td>20.43</td>
<td>-11.58</td>
<td>29.26</td>
<td>-0.52</td>
<td>48.68</td>
<td>19.04</td>
</tr>
<tr>
<td>VIX</td>
<td></td>
<td>27.14</td>
<td>-5.71</td>
<td>28.62</td>
<td>-0.32</td>
<td>49.00</td>
<td>17.53</td>
</tr>
<tr>
<td>Volatility</td>
<td></td>
<td>32.00</td>
<td>-0.70</td>
<td>21.71</td>
<td>-0.19</td>
<td>54.39</td>
<td>13.18</td>
</tr>
</tbody>
</table>

Source: Bloomberg. Weekly CAC40 Index data between January 1st, 1987 and April 30th, 2008. Computations by the authors. The IMS corresponds to the Index of Market Shocks (Maillet and Michel, 2003), the WhIMS to the Wavelet-heterogeneous Index of Market Shocks, and the Volatility corresponds to the one-year daily annualized volatility of returns on the CAC40. The VIX is an aggregate measure of the implied volatility of a wide range of S&P 500 options. All figures – except Sharpe ratios – are expressed in percentages. The column “State” indicates the regime issued from the classification. Frequency represents the percentage of periods in each corresponding state. Mean and volatility represent annualized first and second central conventional moments of the conditional return in the various states. The Sharpe ratio is calculated by subtracting the risk free rate (Eonia) from the rate of return on the portfolio and dividing by the standard deviation of the portfolio returns. Up (Large down) indicates the frequency of positive (large negative) returns in each state conditional samples.
Appendix

Table A1: MSE on the Global Set, Training Set and Test Set

<table>
<thead>
<tr>
<th>Model</th>
<th>Global MSE</th>
<th>Training MSE</th>
<th>Test MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(5)</td>
<td>2.47</td>
<td>2.06</td>
<td>2.82</td>
</tr>
<tr>
<td>MLP(5)</td>
<td>2.46</td>
<td>2.33</td>
<td>2.62</td>
</tr>
<tr>
<td>HMC-MLP1 (5)</td>
<td>2.44</td>
<td>2.27</td>
<td>2.53</td>
</tr>
<tr>
<td>HMC-MLP2 (5)</td>
<td>2.46</td>
<td>2.29</td>
<td>2.61</td>
</tr>
</tbody>
</table>

Source: Bloomberg. Weekly CAC40 Index data from January 1st, 1987 to April 30th, 2008. The prediction performance is measured by the mean squared error (MSE) over three sets of data. The first corresponds to the whole period (5,243 observations), the second to a training set (the 3,670 first observations) and the third to a test period (the last 1,573 data points). HMC-MLP1 corresponds to the MSE criterion whilst HMC-MLP2 is relative to the maximum normalized trace. The maximum number of lags considered is indicated in parentheses. Computations by the authors.

Figure A1: Clustering of the Kohonen Map of States of Market Turbulence (Five-category Tree)

Source: Bloomberg. Weekly CAC40 Index data between January 1st, 1987 and April 30th, 2008. Clusters resulting from a Five-category Hierarchical classification of the four normalized inputs of the Kohonen Map: the WhIMS (first (x,y) coordinate) and the three-state conditional probabilities (the three other (x,y) coordinates, P1, P2 and P3). The Bold line represents the mean Code Vector of each cell. Computations by the authors.